MATHEMATICAL CALCULATION METHOD TO COMPUTE THE SURFACE OF SPHERE UNFOLDED

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Abstract. *The sphere is an undeveloped (unfolded) surface. The surface of the sphere can be developed by approximate methods. This paper presents a parallel calculus between the method of the spherical sector spindles and the method using MATHEMATICA program, for the flat unfold surface of the sphere. The general case is the sphere having radius "r" sectioned by "n" radial sectional plans. The specific case is presented for a three values of sphere radius: $r = 25; 50 ; 100$ mm and $n = 12$ radial sectional plans .*

Keywords: *Sphere developed, flatten unfold, spherical sector method, mathematical computing software*

1. INTRODUCTION

The sphere is an *undeveloped* surface. Its surface can be flatten unfold only by approximate methods. The developing methods, the most common are: the spherical sectors spindles method, the spherical zones method, the pentagon's method and the spherical triangles method.

The most commonly method, of spherical sector spindles, is presented and used in the Descriptive Geometry. The accuracy developing of the sphere surfaces depends on the number of spherical sector spindles.

The applications of the developing of a sphere surface can still be seen when cut bands of sheet, required for covering the spherical bodies in the chemical industry and this technique is like peeling an orange.

Even the Archimedes tomb contained a sculpture illustrating his favorite mathematical demonstration, consisting of a sphere and a cylinder of the same diameter and height.

Archimedes showed first that the ratio between volume and the area of the sphere are same 2/3 of cylinder volume and area respectivley.

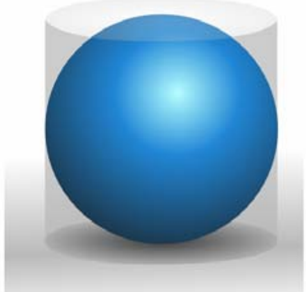

Fig. 1. Sculpture from the tomb of Archimedes

This paper proposes the developing of a sphere surface on a cylinder surface, like the sculpture from the tomb of Archimedes. This paper presents a parallel calculus between the spherical sector spindles method and the mathematical method of calculating the developing of a sphere. The spherical spindle is a portion of the surface of the sphere, which is between two consecutive meridians, and is obtained by cutting the sphere with vertical projection planes.

This case is a general case for a sphere of radius *"r"* and a number of plans *"n"*. The specific calculus case is presented for a sphere of radius $r = 25$ *mm* and a number of plans $n = 12$.

2. SPHERE DEVELOPING BY METHOD OF SPHERICAL SPINDLES

The sphere of Figure 2 having the center in the $\Omega(\omega, \omega')$ point and the radius $r = 25$ mm it is considered, which intersects with twelve equidistant vertical projection planes, passing through the center of the sphere and divide the sphere into twelve spherical spindles.

Due to the similarity developing of these spindles, the spindle developing is determined between the planes *[T1] and [T2],* while for the others do likewise (Figure 2). To develop a spherical spindle, we are considered four auxiliary level planes, $[N_1']$, $[N_2']$, $[N_3']$ and $[N_4']$, went so, the arcs determined on the meridian circle, to be equal to each other: $1'2' = 2'3' = 3'4' = 4'5'$(Figure 2). These plane intersects the sphere by circles, and the considered spindle as circular arcs *lj, mn, pq* and *ab,* which are found in real size in the horizontal projection

The height of a developed spherical spindle is half the length of the meridian circle, that is to say πR. We obtain an approximate sphere developing (Figure 3).

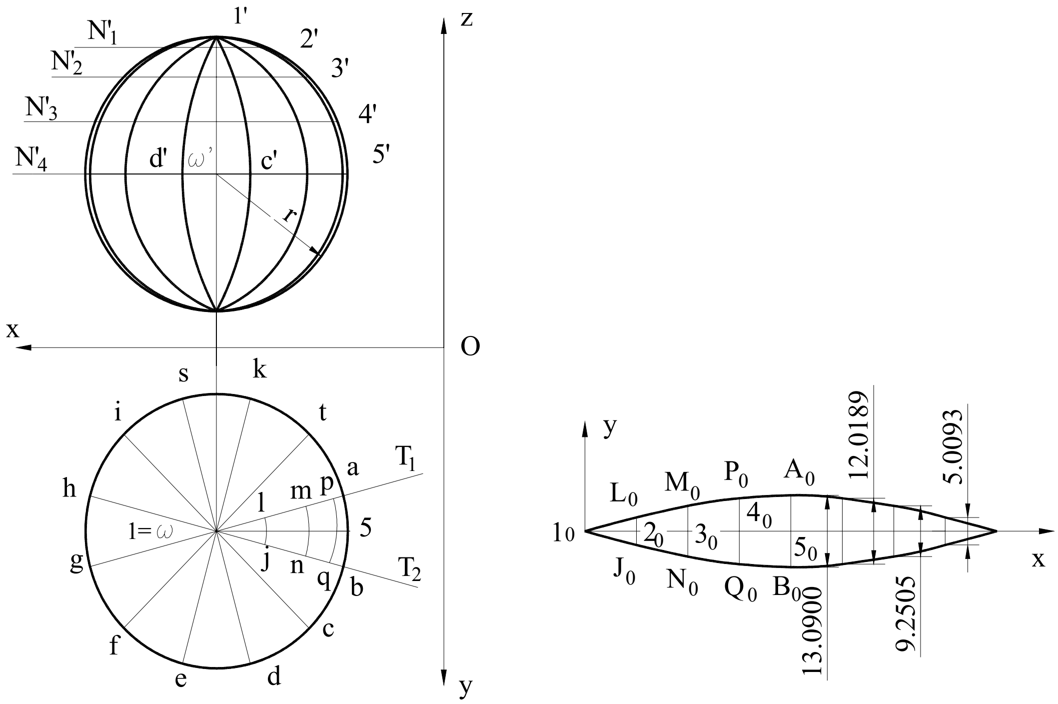

Fig. 2. Developing of a spherical sector spindle

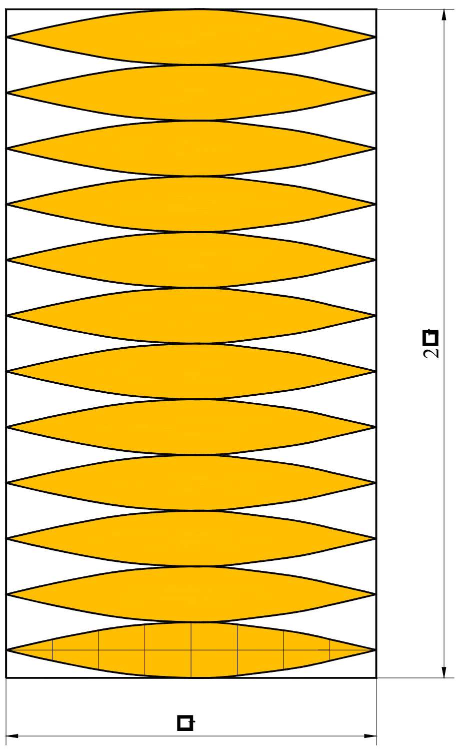

Fig. 3. Developing of the sphere

3. SPHERE R25 DEVELOPIND BY MATHEMATICAL METHOD

The same sphere from Figure 4, of radius *"r"*, which intersects with a number *"n"* of plans project equidistant vertical (for the particular case $r = 25$ mm, $n = 12$ plans) it is considered, passing through its center and divide sphere in twelve spherical spindles.

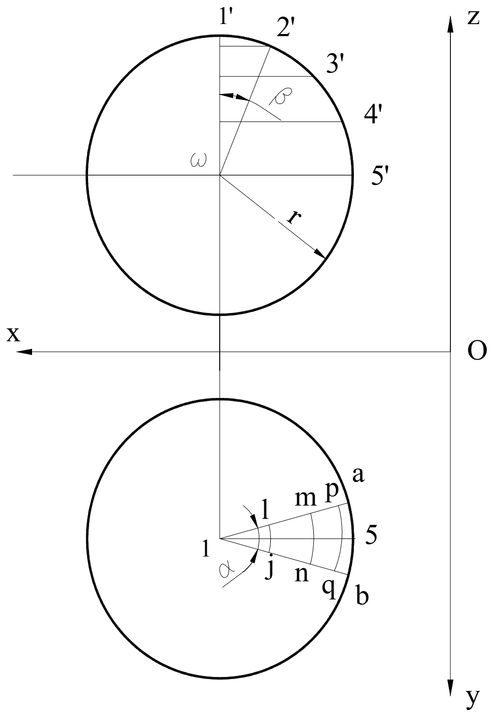

Fig. 4. Mathematical data

$$\widehat{jl} = \alpha \cdot 22' = \frac{2 \cdot \pi}{n} \cdot 22' \qquad (1)$$

$$Sin\beta = \frac{22'}{r} \Rightarrow 22' = r \cdot Sin\beta = r \cdot Sin\frac{\pi}{8} \qquad (2)$$

$$\widehat{jl} = \frac{2 \cdot \pi}{n} \cdot r \cdot Sin\frac{\pi}{8} \qquad (3)$$

The values of following arcs will be obtained:

$$\widehat{mn} = \frac{2 \cdot \pi}{n} \cdot r \cdot Sin\frac{2 \cdot \pi}{8} \qquad (4)$$

$$\widehat{pq} = \frac{2 \cdot \pi}{n} \cdot r \cdot Sin\frac{3 \cdot \pi}{8} \qquad (5)$$

$$\widehat{ab} = \frac{2 \cdot \pi}{n} \cdot r \cdot Sin\frac{4 \cdot \pi}{8} \qquad (6)$$

This will get the points that will pass the chart, and by polynomial interpolation of these points will get the respectively polynomial:

Interpolating nodes of polynomial function for MATHEMATICA program are:

[{{0,0},{π/8·r,(2·π)/n·r/2·Sin[π/8]},{2·π/8·r,(2·π)/n·r/2·Sin [2·π/8]},

{3·π/8·r,(2·π)/n·r/2·Sin[3·π/8]},{4·π/8·r,(2·π)/n·r/2·Sin[4·π /8]},{5·π/8·r,(2·π)/n·r/2·Sin[3·π/8]},

{6·π/8·r,(2·π)/n·r/2·Sin[2·π/8]},{7·π/8·r,(2·π)/n·r/2·Sin[π/8]},{8·π/6·r,0}},x] $\sqrt{2}$

The polynomial function obtained in MATHEMATICA program is:

$f_{(x)}$=732421875 · **π** ^{6} (-2054788736 · Cos[**π** /8]+1955 (609609+339416 · $\sqrt{2}$ -354432 · Sin[**π** /8])) –

-((100·**π**-3·x)x(-156250000·**π**^{5}·x(-7098867776 · Cos[**π**/8]+391(10711701+5472764 · $\sqrt{2}$ -4350528 · Sin[**π** /8]))+

+181250000·**π**^{4} · x^2 (-1651473824 · Cos[**π** /8]+391 (2547545+1172006 · $\sqrt{2}$ -789152 · Sin[**π** /8]))+

+1472000000 · **π** ^{3} x^3 (26416390 · Cos[**π** /8]-17 (957957+403535 · $\sqrt{2}$ -242970 · Sin[**π** /8]))+

+1280000 · **π** ^{2} · x^4 (-2048170124 · Cos[**π** /8]+391 (3280277+1298330 · $\sqrt{2}$ -722540 · Sin[**π** /8])) -

-21299200 · **π** x^5 (-4144448 · Cos[**π** /8]+391 (6699+2552 · $\sqrt{2}$ -1344 · Sin[**π** /8]))+

+65536 · x^6 (-17881864 · Cos[**π** /8]+391 (319 (91+34 · $\sqrt{2}$)-5512 · Sin[**π** /8]))))/(93523911437988281 25 · **π** ^{7})

The graph function, for $r = 25$ mm, $n = 12$, obtained in MATHEMATICA program is as follows în figure 5:

Plot[f(x),{x,0,8 · π/8 ·r},AspectRatio→Automatic,Filling→Axis,AxesLabel→{ " x[mm]","y[mm]"}, PlotRange→{{0,80},{0,7}}]

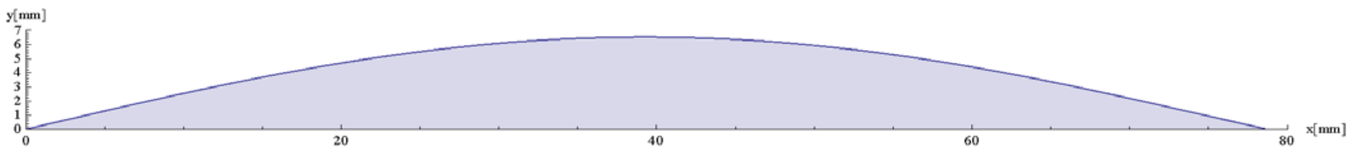

Fig. 5. Graph f(x)function

The finding area of one developed spherical spindle is:

$A_{f25} = 644.4769\ mm^2$,

The total area of the developing of the sphere can be determined, that is $A_{t25} = 12 \times A_{f25} = 7733{,}7228\ mm^2$.

Knowing that sphere area is

$A_{s25} = 4 \times \pi \times 25^2 = 7853{,}9816\ mm^2$

The deviation between the developing sphere area can be determined, being :

$$\delta_{25} = \frac{7853{,}9816 - 7733.7228}{7733.7228} \cdot 100 = 1.555\%$$

4. SPHERE R50 DEVELOPIND BY MATHEMATICAL METHOD

It is considered the sphere from Figure 4, of radius $r=50$ *mm,* which intersects with a number $n=12$ plans project equidistant vertical, passing through its center and divide sphere in twelve spherical spindles.

The finding area of one developed spherical spindle calculate wit respect the same algorithm is:

$A_{t50} = 12 \times A_f = 12 \times 2621{,}5858 = 31.459{,}0296\ mm^2$.

Knowing that sphere area is

$A_{s50} = 4 \times \pi \times 50^2 = 31.416\ mm^2$

The deviation between the developing sphere area can be determined, being :

$$\delta_{50} = \frac{31459{,}0296 - 31416}{31416} \cdot 100 = 1{,}37\%$$

5. SPHERE R100 DEVELOPIND BY MATHEMATICAL METHOD

It is considered the sphere from Figure 4, of radius $r=100$ *mm,* which intersects with a number $n=12$ plans project equidistant vertical, passing through its center and divide sphere in twelve spherical spindles.

The finding area of one developed spherical spindle calculate wit respect the same algorithm is:

$A_{t100} = 12 \times A_f = 12 \times 10477{,}6489 = 125.731{,}7868\ mm^2$.

Knowing that sphere area is

$A_{s100} = 4 \times \pi \times 100^2 = 125.663{,}7\ mm^2$

The deviation between the developing sphere area can be determined, being :

$$\delta_{100} = \frac{125.731{,}7868 - 125.663{,}7}{125.663{,}7} \cdot 100 = 0.54\%$$

6. CONCLUSIONS

- ✓ The accuracy of sphere developing with the spherical sectors spindles method, depends on the number of spindles n and radius of sphere r.
- ✓ using the spherical sectors spindles method, the results obtained are close with exact surface area and decrease with value of radius *r*.
- ✓ using the spherical sectors spindles method, the results obtained are close with exact surface area of sphere and decrease with number of sectional plan *n*.
- ✓ The applications of the developing of a sphere can still be seen when cut bands of sheet, required for covering the spherical bodies in the chemical industry and the like.

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