A NUMERICAL METHOD USED TO STUDY THE KINEMATICS OF A SPATIAL DIFFERENTIAL MECHANISM

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Abstract: *The paper aims to present a numerical method used for kinematics analysis of a spatial differential mechanism. The numerical method is based on writing in matrix form of kinematics dependence equations between rigid solids that make up the system. We finally obtain a system of first order differential equations that integrates using numerical integration methods and we obtain the values of kinematics parameters that characterize the mechanism configuration. In the paper is presented only zero-order kinematics analysis that is positional analysis.*

Keywords: *numerical method, spatial differential mechanism, positional analysis, zero-order kinematics analysis*

1. INTRODUCTION

We consider the spatial differential mechanism shown in the figure below (fig.1). This mechanism has one degree of freedom. Further on, we plan to perform its zero-order kinematics analysis that is to determine the values of the position kinematics parameters that characterize the configuration of the mechanism at a given time.

In order to do this, we will consider a fixed reference frame T(Oxyz) relatively to which we study the movement of the entire mechanical system (mechanism) and four mobile reference frames $T_i(O_ix_iy_iz_i)$ (i=1,..,4) which are integral with each rigid solid that make up the mechanical system.

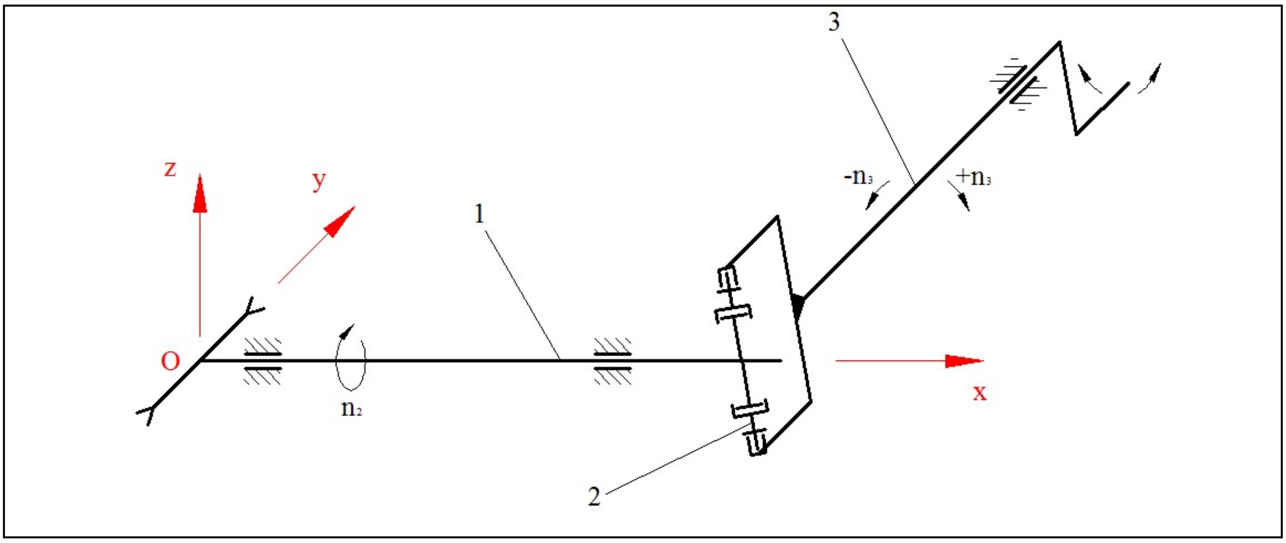

Fig. 1. Spatial differential mechanism

2. WRITING KINEMATICAL EQUATIONS BETWEEN THE RIGID SOLIDS THAT MAKE UP THE SYSTEM

The kinematical relation between the rigid solids "1" and "2" may be written in matrix form in the following way:

$$\{\omega_1\} = [R_1] \cdot \{\omega_2\} \tag{1}$$

In equation (1) the measurements involved have the followings mathematical expressions:

$$\{\omega_1\} = \left[\omega_{x_1} \;\vdots\; \omega_{y_1} \;\vdots\; \omega_{z_1}\right]^T \tag{2}$$

$$\{\omega_2\} = \left[\omega_{x_2} \;\vdots\; \omega_{y_2} \;\vdots\; \omega_{z_2}\right]^T \tag{3}$$

$$[R_1] = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{4}$$

In relation (4), "α" represents the angle between the axes O_1x_1 and O_2x_2 of the reference systems $T_1(O_1x_1y_1z_1)$ and $T_2(O_2x_2y_2z_2)$ respectively. The value of this angle is constant. Vector quantities $\{\omega_1\}$ and $\{\omega_2\}$ are expressed in projections on the axes of the mobile reference frames $T_1(O_1x_1y_1z_1)$ and $T_2(O_2x_2y_2z_2)$.

The kinematical relation between the rigid solids “2” and “3” may be written in matrix form in the following way:

$$[R_2]\cdot\{\omega_3\} = \{\omega_2\} + [R_2]\cdot\{\dot{q}_{23}\} \tag{5}$$

In equation (5) the measurements involved have the followings mathematical expressions:

$$\{\omega_3\} = \left[\omega_{x_3} \mid \omega_{y_3} \mid \omega_{z_3}\right]^T \tag{6}$$

$$\{\dot{q}_{23}\} = \left[0 \mid 0 \mid \dot{q}_{23}\right]^T \tag{7}$$

$$[R_2] = \left[\begin{array}{c|c|c} \cos q_{23} & 0 & -\sin q_{23} \\ \hline 0 & 1 & 0 \\ \hline \sin q_{23} & 0 & \cos q_{23} \end{array}\right] \tag{8}$$

Vector quantities $\{\omega_3\}$ and $\{\dot{q}_{23}\}$ are expressed in projections on the axes of the mobile reference frame $T_3(O_3x_3y_3z_3)$.

The kinematical relation between the rigid solids “3” and “4” may be written in matrix form in the following way:

$$[R_3]\cdot\{\omega_4\} = \{\omega_3\} + [R_3]\cdot\{\dot{q}_{34}\} \tag{9}$$

In equation (9) the measurements involved have the followings mathematical expressions:

$$\{\omega_4\} = \left[\omega_{x_4} \mid \omega_{y_4} \mid \omega_{z_4}\right]^T \tag{10}$$

$$\{\dot{q}_{24}\} = \left[0 \mid \dot{q}_{34} \mid 0\right]^T \tag{11}$$

$$[R_3] = \left[\begin{array}{c|c|c} \cos q_{34} & 0 & -\sin q_{34} \\ \hline 0 & 1 & 0 \\ \hline \sin q_{34} & 0 & \cos q_{34} \end{array}\right] \tag{12}$$

Vector quantities $\{\omega_4\}$ and $\{\dot{q}_{34}\}$ are expressed in projections on the axes of the mobile reference frame $T_4(O_4x_4y_4z_4)$.

The kinematical relation between the rigid solid “1” and the element which is supposed to be fixed (zero element or frame) may be written in matrix form as followings:

$$[A]\cdot[R_{10}]\cdot\{\omega_1\} = \{0\} = [0 \mid 0]^T \tag{13}$$

In equation (13) the measurements involved have the followings mathematical expressions:

$$[A] = \left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \end{array}\right] \tag{14}$$

$$[R_{10}] = \left[\begin{array}{c|c|c} \cos\varphi_{10} & -\sin\varphi_{10} & 0 \\ \hline \sin\varphi_{10} & \cos\varphi_{10} & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \tag{15}$$

where:

$$\dot{\varphi}_{10} = \dot{\Phi}_{z_1} = \omega_{z_1} \tag{16}$$

The kinematical relation between the rigid solid “4” and the element which is supposed to be fixed (zero element or frame) may be written in matrix form as followings:

$$[B]\cdot[R_{40}]\cdot\{\omega_4\} = \{0\} = [0 \mid 0]^T \tag{17}$$

In equation (13) the measurements involved have the followings mathematical expressions:

$$[B] = \left[\begin{array}{c|c|c} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \tag{18}$$

$$[R_{40}] = \left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & \cos\theta_{40} & -\sin\theta_{40} \\ \hline 0 & \sin\theta_{40} & \cos\theta_{40} \end{array}\right] \tag{19}$$

where:

$$\dot{\theta}_{40} = \dot{\Phi}_{x_4} = \omega_{x_4} \tag{20}$$

Equations (1), (6), (9), (13), (16), (17) and (20) form a system of sixteen first order differential equations with sixteen unknowns that may be solved using numerical integration methods and determine the kinematical parameters of the four rigid solids that make up the mechanical system (mechanism). The kinematical study was performed over a period of two seconds. The angle between the axes O_1x_1 and O_2x_2 has the value of ten degrees. The results are shown in figures 2-16. Thus, in figure 1 is shown the variation in relation to time of the self-rotation angle φ_{10} of the rigid solid “1”. Analyzing the figure it may be observed a linear variation of the angle in relation to time which corresponds to a uniform rotation around the axis O_1z_1..

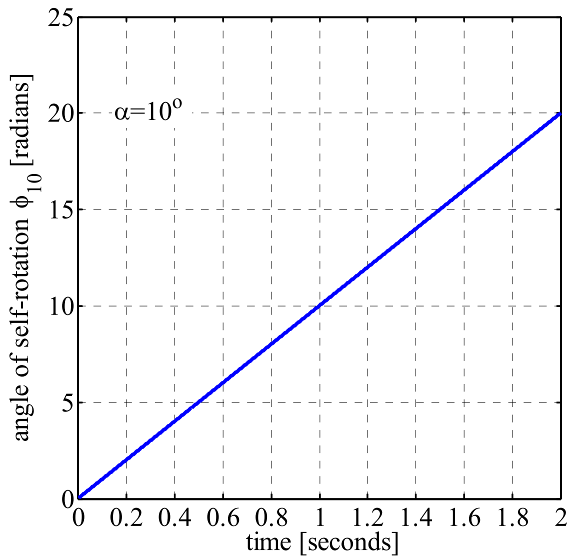

Fig. 2. Self-rotation angle size variation with respect to time

Figure 2 shows the variation in relation to time of the rotation angle around the axis O_1x_1. Analyzing the figure it may be seen that the angle Φ_{x_1} is zero constant value which means that rigid solid “1” has no rotating movement around this axis.

In figure 3 is shown the variation with respect to time of the size of the rotation angle around the axis O_1y_1. Analyzing the figure we see that the angle Φ_{y_1} is zero constant value throughout the movement which means the rigid solid “1” does not rotate around this axis.

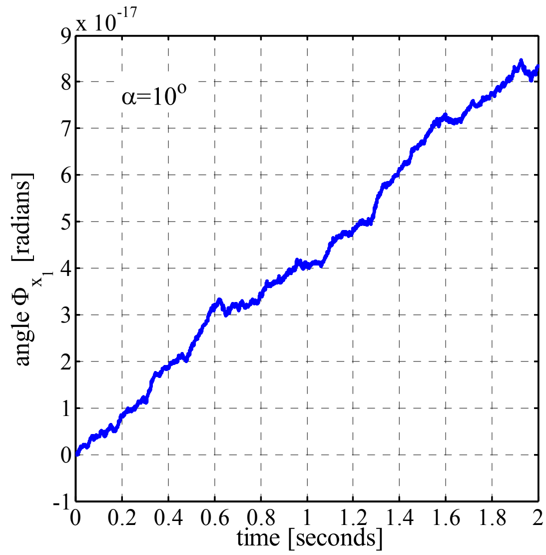

Fig. 3. Variation of angle of rotation around the axis O_1x_1 (Φ_{x_1}) with respect to time

In figure 4 is shown the variation with respect to time of the rotation angle Φ_{z_1} around the axis O_1z_1. Analyzing the figure it may be seen that the size of the angle Φ_{z_1} presents a linear variation with respect to time which is identical with the size variation of the self-rotation angle φ_{10} presented in figure 1.

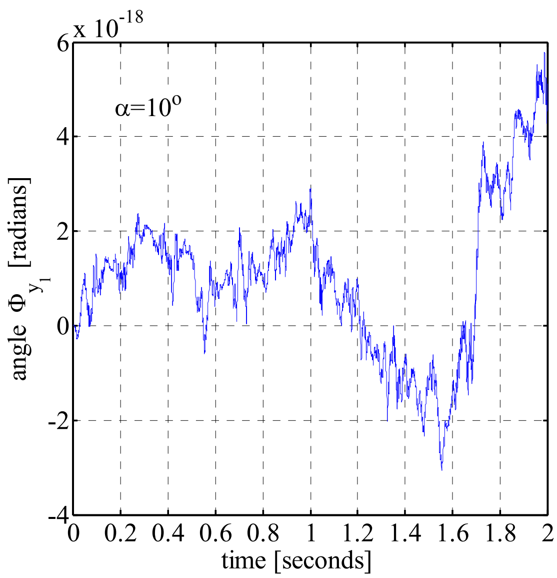

Fig. 4. Variation of angle of rotation around the axis O_1y_1 (Φ_{y_1}) with respect to time

This is explained by the fact that between the sizes of the two angles exists the differential relation (16). It may also be seen that the angle Φ_{z_1} increases continuously throughout the movement

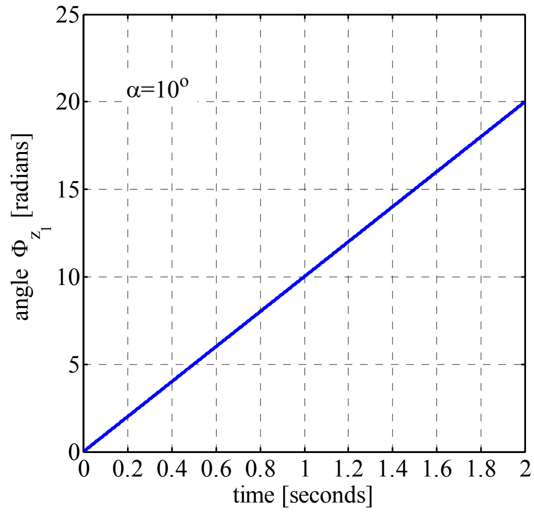

Fig. 5. Variation of angle of rotation around the axis O_1z_1 (Φ_{z_1}) with respect to time

In figure 5 is shown the variation with respect to time of the rigid body “2” rotation angle Φ_{x_2} around the axis O_2x_2. Analyzing the figure it may be seen a linear variation of the size of the angle in relation to time. One can see that the angle Φ_{x_2} increases continuously throughout the movement. In figure 6 is presented the variation with respect to time of the rigid body “2” rotation angle Φ_{y_2} around the axis O_2y_2. Analyzing the figure we see that throughout the movement the angle Φ_{y_2} has a constant value equal to the angle α. Angle “α” is the angle between the axes O_1x_1 and O_2x_2 of the reference systems $T_1(O_1x_1y_1z_1)$ and $T_2(O_2x_2y_2z_2)$ respectively. The angle “α” is ten hexadecimal degrees.

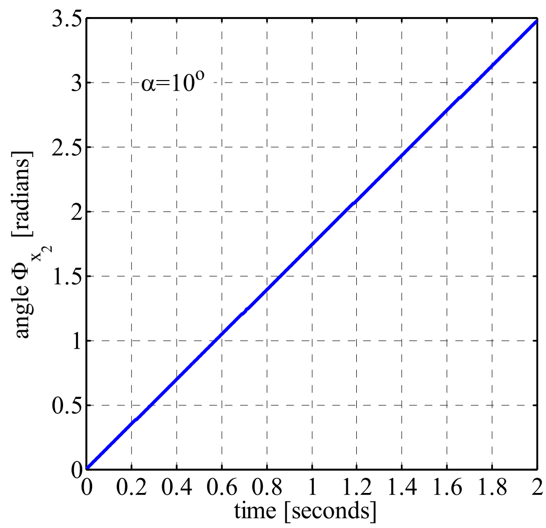

Fig. 6. Variation of angle of rotation around the axis O_2x_2 (Φ_{x_2}) with respect to time

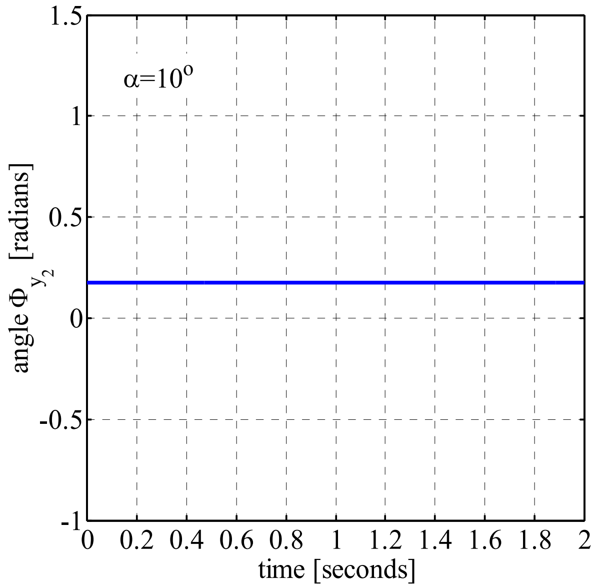

Fig. 7. Variation of angle of rotation around the axis O_2y_2 (Φ_{y_2}) with respect to time

In figure 7 is shown the variation with respect to time of the rigid body "2" rotation angle Φ_{z_2} around the axis O_2z_2. Analyzing the figure it may be seen a linear variation of the size of the angle in relation to time. Comparing the figures 4 and 7 it may be seen a rather great similarity between graphs describing the variations of the two angles. It may also be seen that the angle Φ_{z_2} increases continuously throughout the movement. In figure 8 is shown the variation with respect to time of the rigid body "3" rotation angle Φ_{x_3} around the axis O_3x_3. Analyzing the figure it may be seen a periodic variation between two minimum and maximum limits of the angle size Φ_{x_3} ..

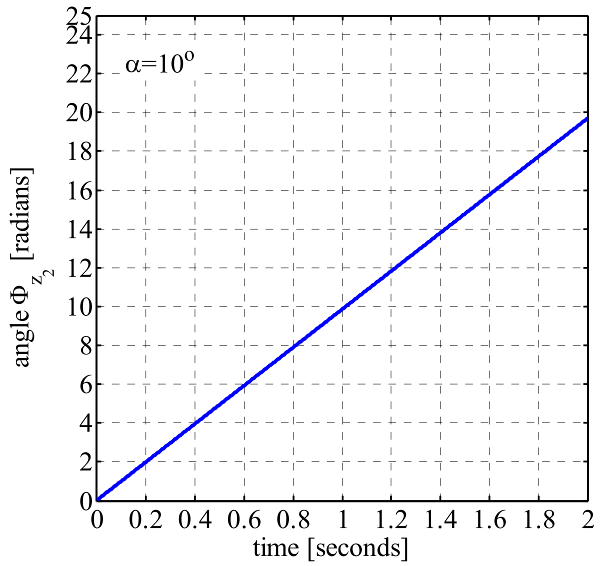

Fig. 8. Variation of angle of rotation around the axis O_2z_2 (Φ_{z_2}) with respect to time

In figure 9 is shown the variation with respect to time of rigid body "3" rotation angle Φ_{y_3} around the axis O_3y_3. Analyzing the figure it may be seen a periodic variation between two minimum and maximum limits of the angle size Φ_{y_3}

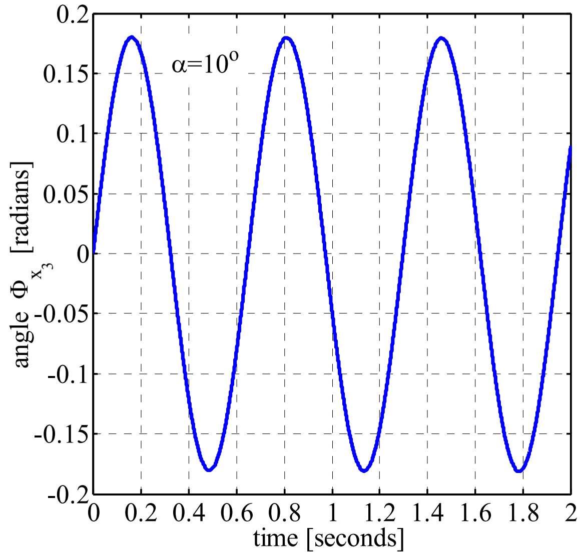

Fig. 9. Variation of angle of rotation around the axis O_3x_3 (Φ_{x_3}) with respect to time

In figure 10 is shown the size variation with respect to time of the rigid solid "3" rotation angle Φ_{z_3} around the axis O_3x_3. Analyzing the figure it may be seen that the angle size variation is not linear. It may also be seen that the angle Φ_{z_2} increases continuously throughout the movement.

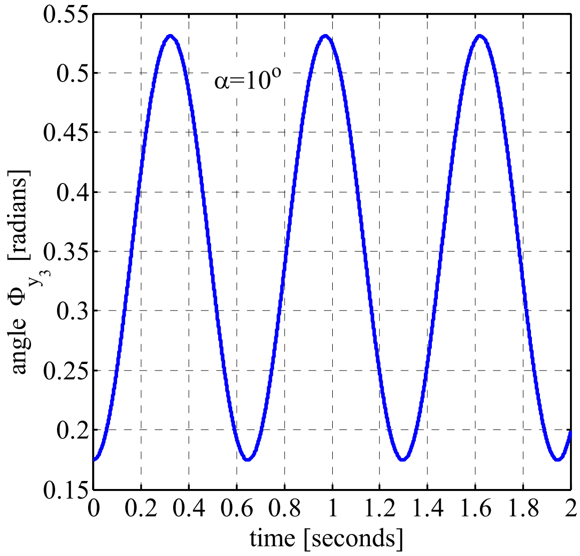

Fig. 10. Variation of angle of rotation around the axis O_3y_3 (Φ_{y_3}) with respect to time

In figure 11 is shown the size variation with respect to time of the rigid solid "4" rotation angle Φ_{x_4} around the axis O_4x_4.

Analyzing the figure it may be seen a periodic variation between two minimum and maximum limits of the angle size Φ_{x_4}.

It should be noted that the angle of rotation Φ_{x_4} is identical to the nutation angle θ_4 of rigid body "4".

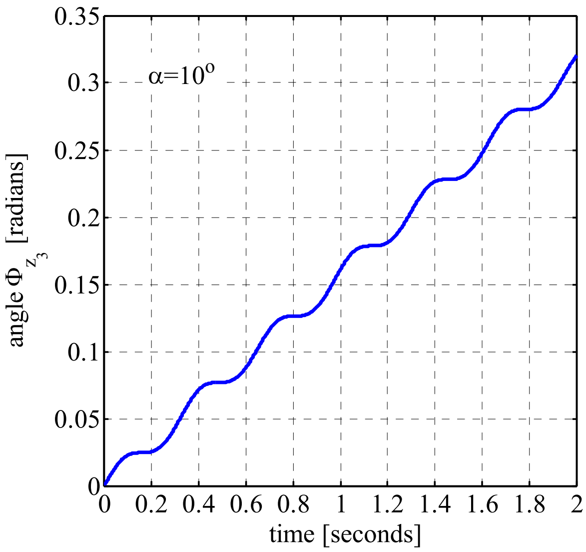

Fig. 11. Variation of angle of rotation around the axis O_3z_3 (Φ_{z_3}) with respect to time

In figure 12 is shown the size variation with respect to time of the rigid solid "4" rotation angle Φ_{y_4} around the axis O_4y_4. Analyzing the figure we see that the angle Φ_{y_4} is zero constant value throughout the movement which means the rigid solid "4" does not rotate around this axis.

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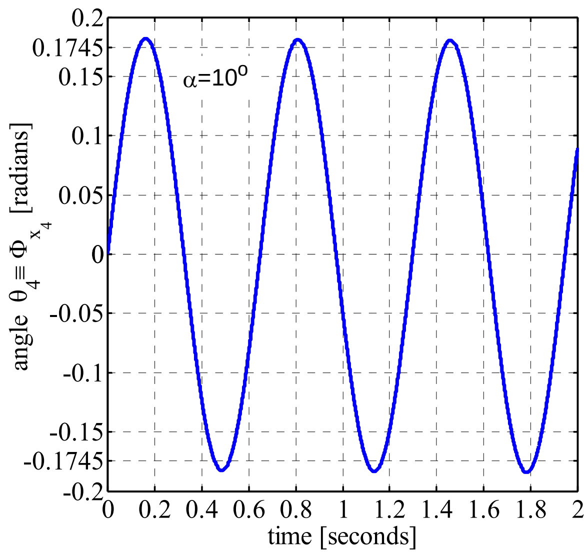

Fig. 12. Variation of angle of rotation around the axis O_4x_4 ($\Phi_{x_4} \equiv \theta_4$) with respect to time

In figure 13 is shown the size variation with respect to time of the rigid solid "4" rotation angle Φ_{z_4} around the axis O_4z_4. Analyzing the figure we see that the angle Φ_{z_4} is zero constant value throughout the movement which means the rigid solid "4" does not rotate around this axis

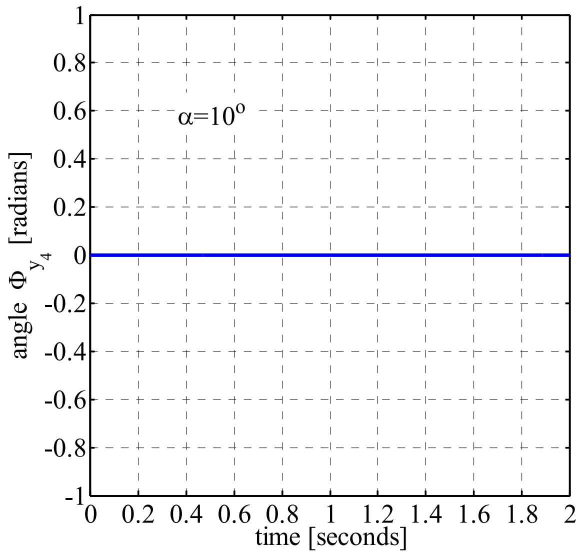

Fig. 13. Variation of angle of rotation around the axis O_4y_4 (Φ_{y_4}) with respect to time

The figure 14 presents the variation with respect to time of the angle describing the relative motion between the rigid solids "2" and "3" of the system. Analyzing the figure it may be seen a linear variation of q_{23} angle size with respect to time. One can also notice a similarity between the appearance of graphs shown in figures 7 and 14 only difference being the sign. One can also see that the q_{23} angle increases continuously throughout the movement.

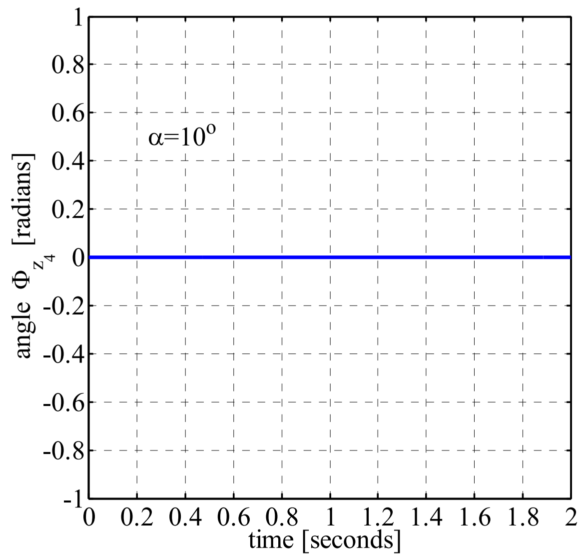

Fig. 14. Variation of angle of rotation around the axis O_4z_4 (Φ_{z_4}) with respect to time

The figure 15 presents the variation with respect to time the variation of the angle describing the relative motion between the rigid solids “3” and “4” of the system. Analyzing the figure it may be seen a periodic variation between two minimum and maximum limits of the angle size q_{34}.

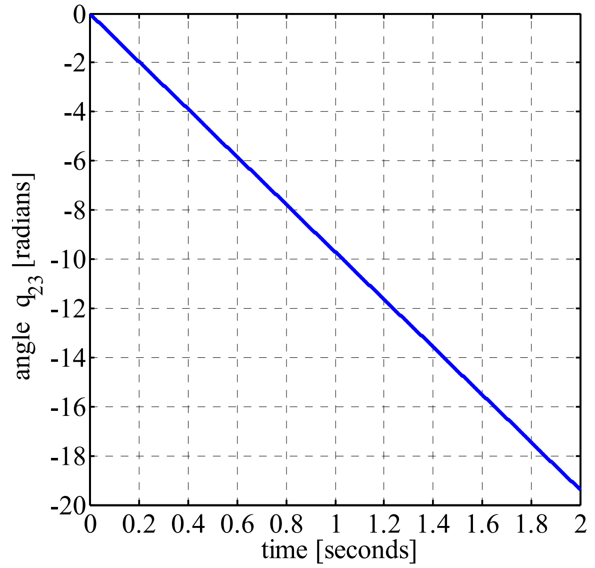

Fig. 15. Variation in relation to time of the relative rotation angle between rigid solids “2” si “3”

One can also notice a resemblance between the graph shape shown in figure 16 and the form of the graphs presented in figures 11, 10 and 9. The initial value of q_{34} angle is equal to the angle value “α” expressed in radians. In all fifteen graphs presented in the paper angular measurements values are expressed in radians.

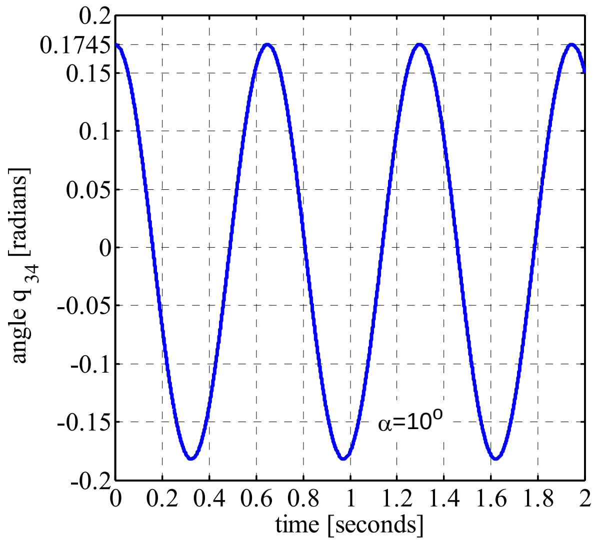

Fig. 16. Variation in relation to time of the relative rotation angle between rigid solids “3” si “4”

3. CONCLUSIONS

In order to perform the study of the mechanical system kinematics were used kinematical equations written in matrix form.

The paper was presented only zero-order kinematical study that is positional analysis. The method can also be extended to first order kinematical study that is speeds kinematical survey.

Numerical method presented in the paper has a high degree of generality it ca be applied to any mechanical kinematical study.

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