TORSIONAL VIBRATIONS DURING THE CONSTANT TORQUE START-UP PHASE OF AN INDUSTRIAL EQUIPMENT (II)

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Abstract: *The stresses in a coupling shaft between a motor and an equipment are caused by vibrations produced by dynamic loading and are superposed on the service stresses. These cumulated stresses have values that sometimes can exceed the allowable values. During the constant torque start-up torsional vibrations occur in the shaft producing a dynamic torque* M_{td} *. This paper shows how to simulate the dynamic response of an equipment to the constant and variable torque start-up and analyzes the influence of the time variation of the torque from zero to the maximum value.*

Keywords: *constant torque start-up, torsional dynamic stresses*

1. THE DYNAMIC RESPONSE OF UNDAMPED ELASTIC SYSTEMS SUBJECTED TO MECHANICAL SHOCKS

Mechanical shocks are short-acting loads, usually of duration comparable with the eigenperiod of the loaded elastic system and are caused by the application of an external torque $M(\tau)$ in a very short time $\Delta\tau$ (Fig. 1).

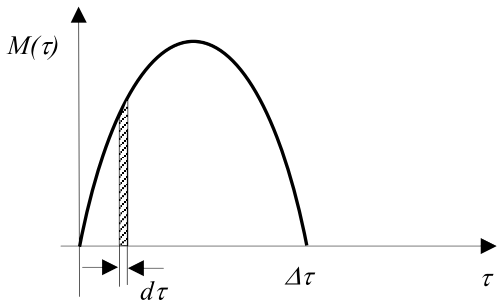

Fig.1. *The mechanical shock with* $\Delta\tau$ *period time*

The mechanical system consisting of a homogeneous wheel with the moment of inertia *J* and an elastic shaft with the stiffness *k*, subjected to the short torque *M(t)*, is considered to study the response of the mechanical undamped systems.

The torque *M(t)* can be considered constant for a very short period of time $\Delta\tau$ (Fig. 1). The area of the hashed surface is equal to an infinitesimal angular momentum *dK*, producing a variation of the angular velocity $\Delta\omega$ according to:

$$dK = J \cdot d\omega = M(\tau) \cdot d\tau \qquad (1)$$

It's obvious that the elementary angular displacement $d\varphi$ of the wheel of wheel of inertia *J* under the action of infinitesimal angular momentum *dK* is given by the relations:

$$d\varphi = \frac{M(\tau) \cdot d\tau}{J \cdot p} \cdot \sin p(t-\tau) \qquad t > \tau \qquad (2)$$

where *p* is the eigen-frequency of the elastic system:

$$p = \sqrt{\frac{k}{J}} \qquad (3)$$

If the effects of the infinitesimal pulses between $\tau=0$ and $\tau=t$ are summed up according to Riemann, or if the elementary angular displacement (2) is integrated, the expression of the total angular displacement $\varphi(t)$ under the action of torque *M(t)* is obtained as *DUHAMEL's integral :*

$$\varphi(t) = \int_0^t \frac{M(\tau)}{J \cdot p} \cdot \sin p(t-\tau) \cdot d\tau \qquad (4)$$

The total angular displacement of the mechanical system $\varphi(t)$ during the shock loading, given by the relation (4) is called *initial dynamic response*.

The total angular displacement of the mechanical system $\varphi(t)$ after the shock loading $(t>\Delta\tau)$ represents the *residual dynamic response* and is a harmonic function:

$$\varphi(t) = a \cdot \sin pt + b \cdot \cos pt \qquad (5)$$

The constants *a* and *b* in equation (5) can be determined using the boundary conditions for angular displacements and velocities at the time between the initial and residual dynamic response.

2. NUMERICAL SIMULATION OF THE DYNAMIC RESPONSE OF THE MECHANICAL SYSTEM SUBJECTED TO A CONSTANT TORQUE

The particular case of an undamped mechanical system under the action of a constant torque (Fig. 2):

$$M(\tau)=M_0 \tag{6}$$

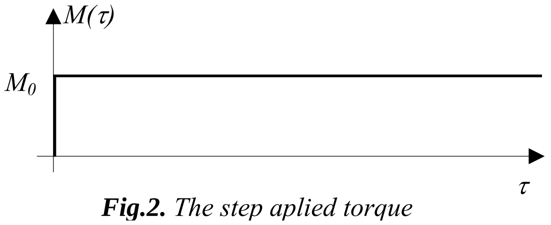

Fig.2. *The step aplied torque*

The variation of the angular displacement *φ(t)* during the application of the constant torque is obtained using the general relation (4):

$$\varphi(t)=\frac{M_0}{J\cdot p^2}\left(1-\cos pt\right) \tag{7}$$

The static angular displacement under the action of the constant torque M_0 is given by:

$$\varphi_{st}=\frac{M_0}{k_t}=\frac{M_0}{J\cdot p^2} \tag{8}$$

The dynamic multiplier of the displacement is defined as the ratio of the angular displacement (7) over the static angular displacement (8):

$$\Psi(t)=\frac{\varphi(t)}{\varphi_{st}}=1-\cos pt \tag{9}$$

In Fig. 3 the variation of the *dynamic multiplier of the displacement* is plotted using MATHCAD for the following particular parameters:

$$J=1kg\cdot m^2;\quad k=4\pi^2\ kg\cdot m^2\cdot s^{-2};\quad M_0=10N\cdot m \tag{10}$$

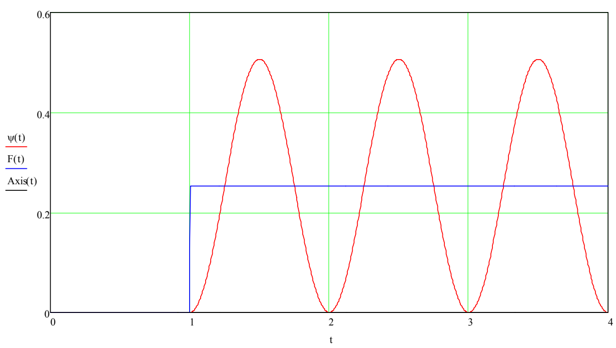

Fig.3. *Dynamic multiplier of the displacement*

Note: The maximum dynamic multiplier of the displacement is $\psi = 2$ and the minimum is $\psi = 0$.

The eigen-frequency *p* depends on the dynamic properties of the mechanical system: the stiffness of shaft *k* and the moment of inertia of the wheel *J*.

$$p=\sqrt{\frac{k}{J}} \tag{11}$$

The same results were obtained by simulating the dynamic response of the undamped mechanical system using MATLAB SIMULINK.

Fig. 4.a shows the block diagram of the undamped system under the action of a constant torque.

The differential equation of the model is:

$$\ddot{\varphi}+\frac{k}{J}\cdot w=\frac{M(t)}{J} \tag{12}$$

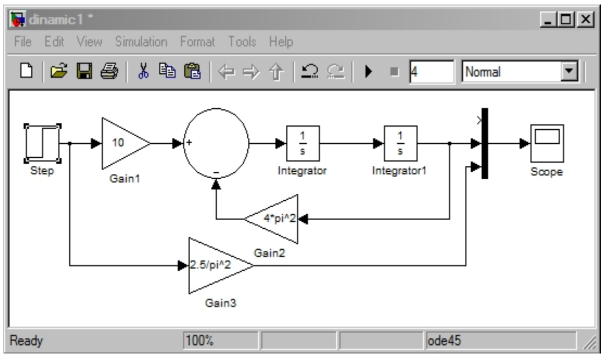

Fig.4.a. *The block diagram of the simulated system*

The variation of the *dynamic multiplier of the displacement* given by differential equation (12) is plotted using MATLAB SIMULINK for the same particular values of parameters (10).

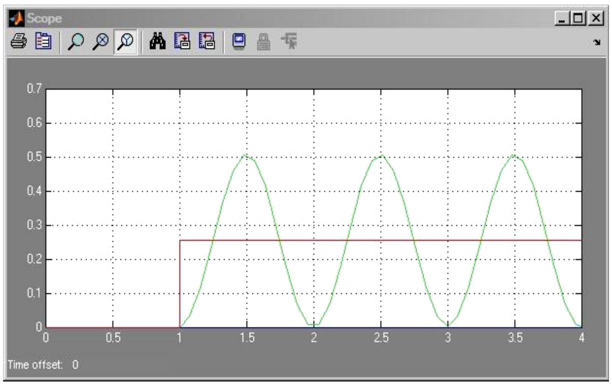

Fig.4.b. *Dynamic response of the undamped system*

The dynamic response of the damped mechanical system can be obtained in the same way.

Figure 5.a shows block diagram of the damped system under the action of the constant torque.

The differential equation of the model is:

$$\ddot{\varphi}+\frac{c}{J}\cdot\dot{\varphi}+\frac{k}{J}\cdot\varphi=\frac{M(t)}{J} \tag{13}$$

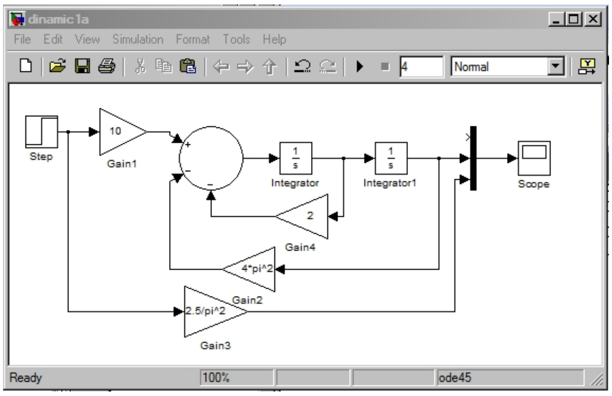

Fig.5.a. *The block diagram of the simulated system*

Fig. 5.b. shows the *dynamic response* given by differential equation (12) for the particular values of parameters:

$$J = 1 kg \cdot m^2; \quad c = 2\ kg \cdot m^2 \cdot s^{-1}$$
$$k = 4\pi^2\ \ kg \cdot m^2 \cdot s^{-2}; \quad M_0 = 10 N \cdot m \qquad (14)$$

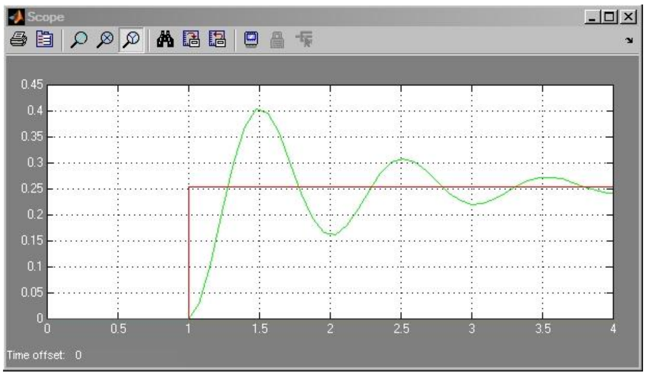

Fig.5.b. *Dynamic response of the damped system*

3. NUMERICAL SIMULATION OF THE DYNAMIC RESPONSE OF THE MECHANICAL SYSTEM SUBJECTED TO A LINEAR INCREASING TORQUE

The case of a linearly increasing torque $M(\tau)$ acting on the undamped mechanical system will be investigated. The variation of the applied torque (Fig. 6) is given by the following relation:

$$M(\tau) = \begin{cases} M_0 \cdot \dfrac{\tau}{t_1} & for \quad 0 < \tau < t_1 \\ M_0 & for \quad \tau > t_1 \end{cases} \qquad (15)$$

Relations (4) and (5) can be used to evaluate the dynamic response of the system during the application of the variable torque ramp:

- for the first time interval $0 < t < t_1$:

$$\varphi(t) = \frac{M_0}{J \cdot p} \int_0^t \frac{\tau}{t_1} \cdot sin\ p(t - \tau) \cdot d\tau \qquad (16)$$

- for the second time interval $t > t_1$:

$$\varphi(t) = a \cdot sin\, pt + b \cdot cos\, pt + \frac{M_0}{J \cdot p^2} \qquad (17)$$

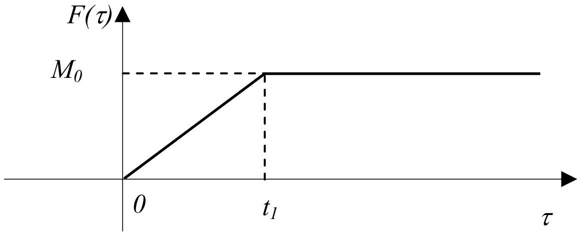

Fig.6

Integrating and substituting the boundary conditions, the angular displacement and velocity are:

- for the first time interval $0 < t < t_1$:

$$\begin{cases} \varphi(t) = \dfrac{M_0}{J \cdot p^2} \left(\dfrac{t}{t_1} - \dfrac{sin\ pt}{pt_1} \right) \\ \dot{\varphi}(t) = \dfrac{M_0}{J \cdot p^2} \cdot \left(\dfrac{1 - cos\ pt}{t_1} \right) \end{cases} \qquad (18)$$

- for the second time interval $t > t_1$:

$$\begin{cases} \varphi(t) = a \cdot sin\, pt + b \cdot cos\, pt + \dfrac{M_0}{J \cdot p^2} \\ \dot{\varphi}(t) = a \cdot p \cdot cos\, pt - b \cdot p \cdot sin\, pt \end{cases} \qquad (19)$$

The constants *a* and *b* were determined from the boundary conditions between two subintervals for both the angular displacements and velocities:

$$\begin{cases} a = -\dfrac{M_0}{J \cdot p^2} \left(\dfrac{1 - cos\ pt_1}{pt_1} \right); \\ b = -\dfrac{M_0}{J \cdot p^2} \left(\dfrac{sin\ pt_1}{pt_1} \right) \end{cases} \qquad (20)$$

Figures 7.a-d show the *dynamic response* given by relations (18) and (19) for the same particular values of the parameters (10):

$$J = 1 kg \cdot m^2; \quad k = 4\pi^2\ \ kg \cdot m^2 \cdot s^{-2};$$
$$M_0 = 10 N \cdot m \qquad (21)$$

and for following values of the torque ramp duration:

t_1=0,1·T; t_2=0,5·T; t_3=T; t_4=1,2·T (22)

where $T = \dfrac{2\pi}{p} = 1\ s$

is the eigen-period of vibration

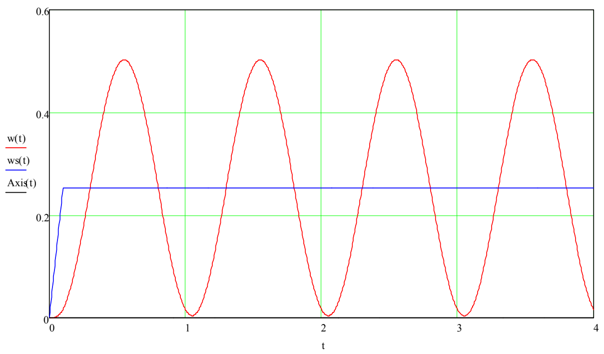

***Fig.7.a**. Dynamic multiplier of the displacement for t_1=0,1T*

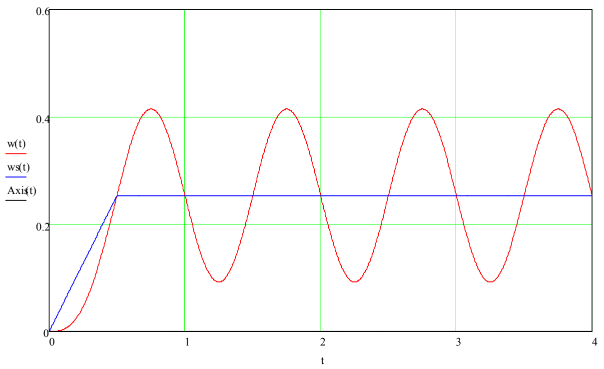

***Fig.7.b.** Dynamic multiplier of the displacement for t_2=0,5T*

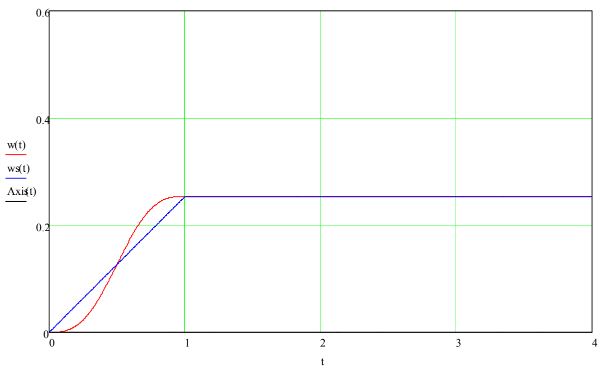

***Fig.7.c.** Dynamic multiplier of the displacement for t_3=T*

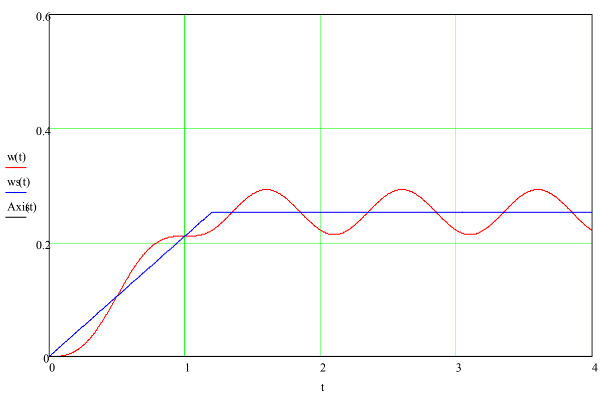

***Fig.7.d.** Dynamic multiplier of the displacement for t_4=1,2T*

Important note: Figure 7.c shows that for the ramp duration $t_1 = T$ the residual dynamic response is missing.

Figure 8.a shows the block diagram of the undamped system subjected to the constant torque.

The differential equation of the model is given by the previous relation (12) .

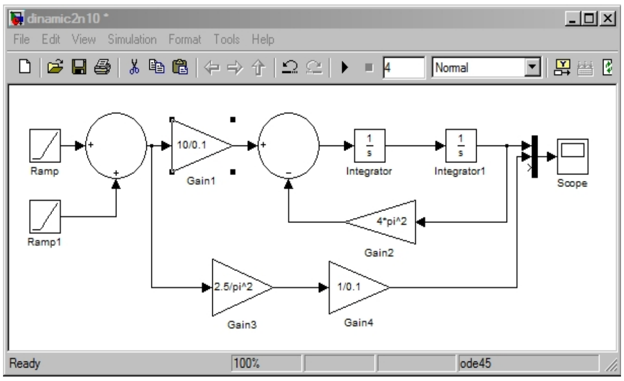

***Fig.8.a.** The block diagram of the simulated undamped system*

Fig. 8.b-e show the *dynamic response* given by differential equation (12) for the same particular values of parameters (10):

$$J = 1kg \cdot m^2; \quad k = 4\pi^2 \ kg \cdot m^2 \cdot s^{-2};$$
$$M_0 = 10N \cdot m$$

and for following values of the ramp duration:

$t_1=0,1 \cdot T; \ t_2=0,5 \cdot T; \ t_3=T; \ t_4=1,2 \cdot T$ (23)

where $T = \dfrac{2\pi}{p} = 1 \ s$ is the eigen-period of vibration.

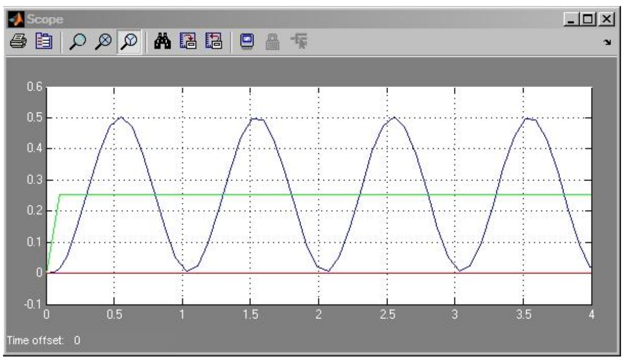

***Fig.8.b.** Dynamic response of the undamped system for t_1=0,1T*

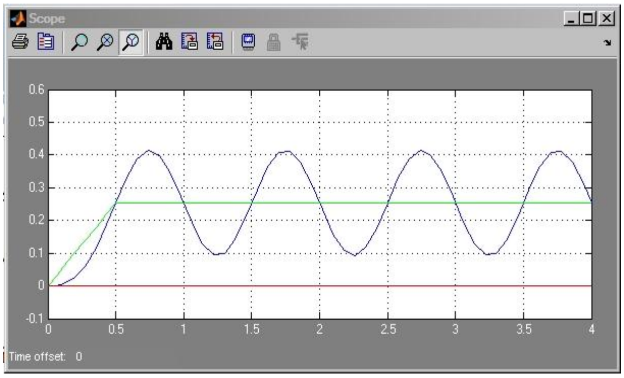

Fig.8.c. *Dynamic response of the undamped system for $t_2=0{,}5T$*

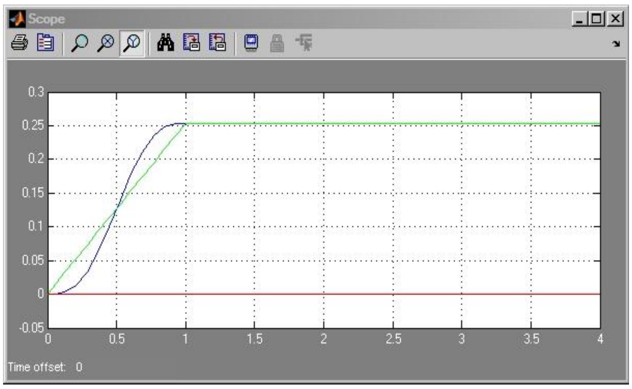

Fig.8.d. *Dynamic response of the undamped system for $t_3=T$*

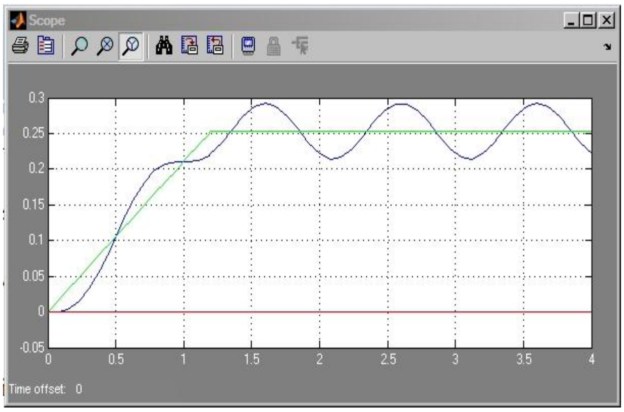

Fig.8.e. *Dynamic response of the undamped system for $t_4=1{,}2\ T$*

The dynamic response of the damped mechanical system can be obtained in a similar manner using MATLAB SIMULINK.

Fig. 9.a shows the block diagram of the simulated damped system subjected to constant torque. The differential equation of the model is:

$$\ddot{\varphi}+\frac{c}{J}\cdot\dot{\varphi}+\frac{k}{J}\cdot\varphi=\frac{M(t)}{J} \qquad (24)$$

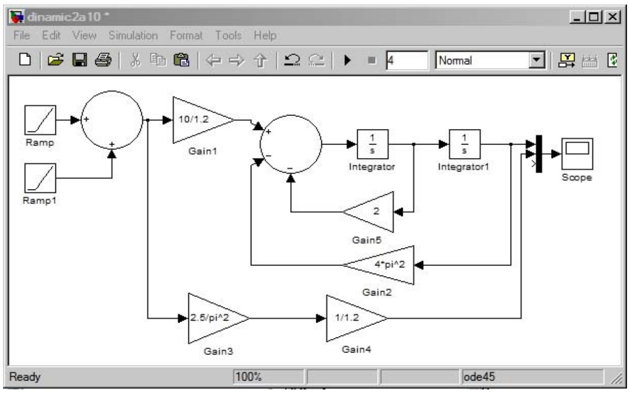

Fig.9.a. *The block diagram of the simulated damped system*

Figures 9.b-e show the the *dynamic response* given by differential equation (12) for the particular values of the parameters:

$$J=1kg\cdot m^2;\quad c=2\ kg\cdot m^2\cdot s^{-1}$$
$$k=4\pi^2\ \ kg\cdot m^2\cdot s^{-2};\quad M_0=10N\cdot m \qquad (25)$$

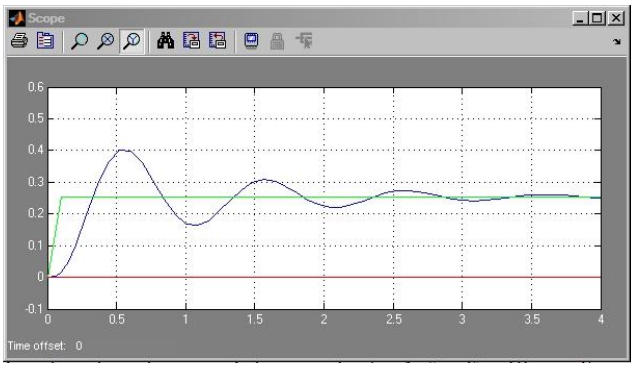

Fig.9.b. *Dynamic response of damped system for $t_1=0{,}1T$*

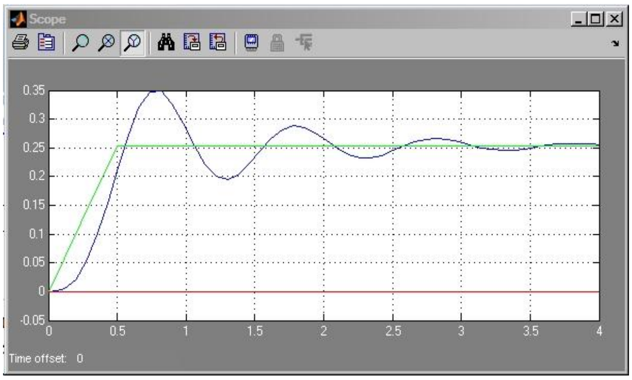

Fig.9.c. *Dynamic response of the damped system for $t_2=0{,}5T$*

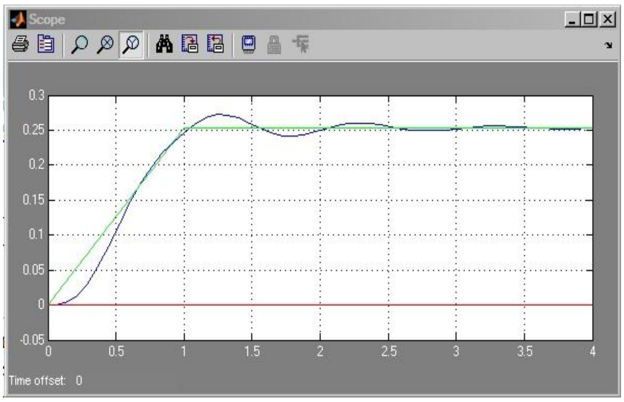

Fig.9.d. *Dynamic response of the damped system for $t_3=T$*

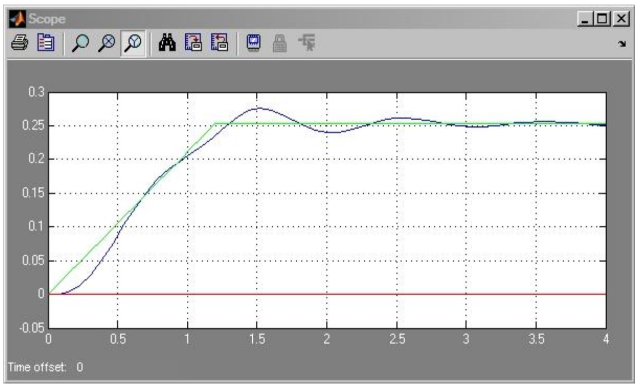

Fig.9.e. *Dynamic response of the damped system for $t_4=1,2\ T$*

4. CONCLUSIONS

The following cases were analyzed using MATLAB SIMULINK:

- Case 1: $\frac{M_s}{2} \leq M_m < M_s$, when the electro-mechanical system remains at rest and the rotor vibrates due to the sudden application of the torque M_m.

- Case 2: $M_m = M_s$, when the electro-mechanical system remains at rest and the rotor vibrates (no rotation or solid body motion occurs) due to sudden application of the torque M_m.

- Case 3: $M_m > M_s$, when the electro-mechanical system moves as a solid body but also experiences torsional vibration due to sudden application of torque M_m.

The simulations performed in MATCAD and MATLAB SIMULINK show the same response of the electro-mechanical system.

The technical solution for the reduction or the total elimination of the dynamic torque variation is the linear application of the driving torque M_m on a time interval equal to the eigen-period of the system.

REFERENCES

[1] Posea, N - *Calculul dinamic al structurilor.* Ed. Tehnică, Bucureşti 1991

[2] Bratu, P.P.- *Vibraţiile structurilor mecanice.* Editura Tehnică, Bucureşti, 2000

[3] Bratu, P.P. - *Izolarea şi amortizarea vibraţiilor la utilaje de construcţii*. Editura INCERC, Bucureşti, 1982

[4] Bratu, P.P., Marin., C. - *Asupra variaţiei coeficientului de încărcare - dinamică Ψ a unui sistem mecanic cu pornire sub sarcină* – ROPET, Petroşani, 2003

[5] Marin, C - *Vibraţiile structurilor mecanice.* Editura IMPULS, Bucureşti, 2003

[6] Marin, C, Bratu, P. - *Dynamic stresses occurring at the start under loading of a dynamical system with constant initial torque,* CONFERENCE on MULTIBODY SYSTEMS' DYNAMICS CDSM - Piteşti, 2007

[7] Marin, C, Vasile Gh. – *Tehnici de modelae si simulare în ingineria mecanică*– Editura Bibliotheca, Târgovişte 2011.