## **DESIGN AND CONTRUCTION OF A MECHATRONIC SYSTEM WITH PHOTOELECTRIC INCREMENTAL TRANSDUCER FOR HIGH PRECISION MINI-DIMENSIONAL MEASUREMENTS**

## **Iulian ILIE<sup>1</sup> , Gheorghe I. GHEORGHE<sup>1</sup>**

<sup>1</sup>National Institute of Research and Development for Mechatronics and Measurement Technique - Bucharest, Pantelimon Road 6-8, District 2, Bucureşti, România, E-mail: incdmtm@incdmtm.ro

*Abstract. This paper presents the design and construction of a mechatronic system with high precision photoelectric incremental sensor for mini-dimensional measuring, system that underlies the construction of mechatronic and adaptronics equipments used for high precision measurement of linear movements.* 

*Keywords: transducer, mechatronics, mini-dimension* 

## **1. INTRODUCTION**

The design and construction of mechatronic system with high precision photoelectric incremental sensor for minidimensional measuring is based on fig. 2., where:







**Figure 2. Kinematic scheme of incremental photoelectric transducer**

#### Legend:

- $S_i$  = input (displacement X of sensor);
- 1 measuring rod;
- 2 incandescent lamp;
- 3 condenser lens;
- 4 incremental vernier;
- 5 incremental dividing ruler;

6 • photocell system;  $S_e$  - output;  $S_e = \{i_A, i_B\}$  = quasisinusoidal alternating current shifted with 2  $\frac{\pi}{6}$ , S<sub>i</sub> = input (alternative to S<sub>1</sub>), (displacement x<sub>1</sub> of cremaillere 7).

If r is the dividing radius of the wheel 8 and R the radius of the wheel 9, x1 causes rotation ϕ with:

$$
\varphi = \frac{x_1}{r};\tag{1}
$$

The sensor is moving with:

$$
x = r \cdot \varphi = \frac{R}{r} x_1; \tag{2}
$$

So: 
$$
Si = \frac{R}{r} S'_{2};
$$
 (3)

Regarding figure 3. is considered: lighting beam emitted by the filament lamp (1) is converted into the parallel beam by the condenser lens (2); is normal on vernier (3); its useful area (one of the 4 networks) "cut" from the incident beam one part that crossing the divider network (4), then is on the photoelement surface (5).

The incandescent lamp emits in  $4\pi$  srad a radiant flux

$$
\phi_{i\boldsymbol{\cdot}}
$$



### **Figure 3. Light beam chart**

Lens emergent flux is:

$$
\varphi_{i}^{\prime} = \tau_{\Theta} \cdot \varphi_{i};\tag{4}
$$

where:  $* \tau_e$  = lens transmittance;

$$
\boldsymbol{\tau}_e = \big(1-\rho_e\big)^m \cdot \big(1-\alpha_e\big)^d, \ \ \text{unde : } \rho_e = \left(\frac{n-1}{n+1}\right)^2; (5)
$$

where:  $* n =$  relative reference index glass – air;  $\rho_e$  = reflectance; n = n (λ) take into account λ, corresponding to maximum sensitivity of photoelement.

∗ m = number of diopters (m= 2);

 $* \alpha_e$  = specific loss due to absorbtion for a geometric path of 1mm (for glass chrome,  $\alpha_e = 0.00048$ );

∗ d = average geometric path traversed by the beam of light in the lens (mm). It is the rays that will incide in the area Sv of vernier.

Emergent flux of the 2 networks is:

$$
\varphi_{\Theta} = \tau_{r} \cdot \varphi_{i}^{\prime} = \tilde{\tau} \cdot \varphi_{i} ; \qquad (6)
$$

where:  $*\tau_r$  = networks transmittance

 $\tilde{\tau}$  = transmittance of the lens-networks system

$$
\tau_r = \frac{1}{2} (1 - \rho_r)^m \cdot (1 - \alpha_r)^d \cdot (1 - \eta) \tau(x)
$$
 (7)

where: ∗η= active surface of vernier active and illuminated surface of pfotoelement is

influenced by parallel misalignment / eccentricity in mounting supports of incremental network and elements settings. For establishing η variation of illumination is neglected in photoelement plan duet o the relative motion of the networks.

 $* \tau$  (x) = transmittance component determined solely by the relative motion of networks.

Constant  $\frac{1}{2}$  $\frac{1}{6}$  is introduced assuming that the ratio between the width of an opaque and one transparent increment is 1.

Noting transmittance components that do not depend on x, with  $\tau_0$  we have:

$$
\tilde{\tau}(x) = \tau_0 \cdot \tau(x) ; \qquad (8)
$$

Irradiance resulting on active surface of photoelement is:

$$
E = \frac{\varphi_e}{S_f};
$$
 (9)

but: 
$$
\varphi_{\Theta} = \tilde{\tau}(x) \cdot \varphi_i;
$$
 (10)

$$
SO: \qquad E = E(x) \tag{11}
$$

Flask current supplied by the pfotoelement varies linearly with irradiance (fig. 4) according to the catalog characteristics

$$
I_{sc} = K . E; \tag{12}
$$



**Figure 4.**

where:  $K = \{g \theta; \theta = \text{characteristic slope};\}$ 

$$
\mathbf{K}_{j} = \mathbf{K}_{0} \cdot \mathbf{S}_{f;}
$$
 (13)

where:  $K_0$  = material constant

so: 
$$
I_{sc} = K_0 \cdot S_f \times E = K_0 \cdot S_f \cdot \frac{\varphi_e}{S_f}
$$
; (14)

$$
I_{sc} = K_0 \cdot \phi_{e};\tag{15}
$$

Flask current does not depend on the area photoelement, only on radiant flux. For an ideal source and a stationary positions of the two networks φe is constant in any normal section on the optical axis of the lens.

$$
I_{sc}(x) = K_0 \cdot \phi_i \cdot \tilde{\tau}(x); \qquad (16)
$$



#### **Figure 5**

Referring to fig. 5., is considered:

• networks 
$$
\begin{cases} A_1/A_2 \\ B_1/B_2 \end{cases}
$$
 are mutual dephased with P/2,

resulting transmittance  $τ/1 - τ$ ;

• networks  $A_1 / B_1$  are mutual dephased with P/4;

Further referring to group of networks and photoelements  $A_1/A_2$ .

Transmittance graph  $\tau_A = \tau_A(x)$  (fig.6.).

$$
T_A: \left[-\frac{P}{2}, \frac{P}{2}\right] \to R;
$$
\n
$$
---\frac{1}{2} \underbrace{\sqrt{\alpha_A}}_{-\frac{P}{P_2}} --- \underbrace{\qquad (17)}
$$
\n
$$
---\frac{1}{2} \underbrace{\sqrt{\alpha_A}}_{-\frac{P}{P_2}} --- \underbrace{\qquad (17)}
$$



$$
\tau_{\mathbf{A}}(x) = \begin{cases} \frac{2}{p}x + 1, & -\frac{p}{2} \le x < 0\\ -\frac{p}{2}x + 1, & 0 \le x \le \frac{p}{2} \end{cases}
$$
(18)

The currents produced by fotoelements A (respectively B) are introduced into the operational amplifier element from electronic unit for signal processing of the transducer, according to the scheme (fig. 7). The first theorem of Kirchhoff's gives the current:

$$
I_A = I_{\rm scl} - I_{\rm sc2};\tag{19}
$$

where:

$$
I_{\rm sc1} = K_0 \cdot \phi_i \cdot \tau_0 \cdot \tau_A(x) \tag{20}
$$

$$
\mathbf{I}_{\text{sc2}} = \mathbf{K}_0 \cdot \phi_i \cdot \tau_0 \left[ (1 - \tau_A(x)) \right];\tag{21}
$$

It was assumed that the two photoelements are identical and identically illuminated. Fig. 8. indicates how current Isc1 and Isc2 are compounds

$$
i_{\mathbf{A}}(x) = K_0 \cdot \varphi_i \cdot \tau_0 \left[ 2\tau_{\mathbf{A}}(x) - 1 \right];\tag{22}
$$



**Figure 7** 



**Figure 8.** 

With the notations of the previous figure:

$$
u = U_r + \frac{R_1}{R_2} \cdot i \tag{23}
$$

$$
u_{A}(x) = K_{0} \cdot \varphi_{i} \cdot \tau_{0} \cdot \frac{R_{1}}{R_{2}} [2\tau_{A}(x) - 1] + U_{R}
$$
 (24)

We decompose  $\tau_A(x)$  in Fourier series in J 1  $\left[-\frac{p}{2}+\frac{p}{2}\right]$ p 2 p and it is:

$$
T_A(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos \frac{2n\pi}{p} \cdot x ;
$$
 (25)

 $b_n = 0$  because  $\tau_A$  is symmetric function.

$$
a_0 = \frac{2}{P} \cdot \frac{2}{\int_{P}^{P} \tau_A(x)dx} = \frac{4}{P} \cdot \frac{2}{\int_{P}^{P} \tau_A(x)dx} = \frac{4}{P} \cdot \frac{2}{\int_{P}^{P} \tau_A(x)dx} = 1
$$
  

$$
\frac{P}{2}
$$
  

$$
\frac{P}{2}
$$
  

$$
a_0 = \frac{4}{P} \cdot \frac{2}{\int_{P}^{P} \tau_A(x) \cos \frac{1}{2}x \cdot dx} = \frac{4}{P} \cdot \frac{2}{\int_{(-\frac{2}{P}x + 1) \cos \frac{1}{2}x \cdot dx} \cdot dx} = \frac{4}{P} \cdot \frac{2}{\int_{P}^{P} \tau_A(x) \cdot dx} = \frac{4}{P} \cdot \frac{2}{\int
$$

$$
\tau_{\mathbf{A}}(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2 \pi^2} \cdot \sin^2 \frac{n\pi}{2} \cdot \cos \frac{2n\pi}{p} \cdot x \tag{26}
$$

But:

l

$$
\sin^2 \frac{n\pi}{2} = \begin{cases} 0, & n-2k \\ 1, & n = 2k+1 \end{cases};
$$
  
\n
$$
\tau_A(x) = \frac{1}{2} + \frac{4}{\pi^2} \cdot \cos^2 \frac{2\pi}{2} + \frac{4}{\pi^2} \cdot \cos^2 \frac{6\pi}{2} + \dots = \frac{1}{2} + \frac{4}{\pi^2} \cdot \frac{6}{\pi^2} \cdot \frac{1}{\pi^2} \cdot \cos \frac{2(2n+1)\pi}{2} \cdot \frac{1}{\pi^2}
$$
\n(27)

#### For networks / photoelements group,  $\tau_B$  (x) is presented as in fig. 9 and its development in Fourier series is made using function f :





$$
\tau_{\mathbf{B}}(x) = f(x) + \frac{1}{2};\tag{28}
$$

f is odd function

 $\mathbf{D}$ 

P

$$
f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin \frac{2n\pi}{p} x ;
$$
 (29)

P

$$
b_n = \frac{4}{p} \int_0^{\frac{\pi}{2}} f(x) \sin \frac{2n\pi}{p} x \cdot dx = \frac{4}{p} \int_0^{\frac{\pi}{2}} \left( \frac{2}{p} \cdot x \cdot \sin \frac{2n\pi}{p} x \right) dx +
$$

$$
+\int_{\frac{P}{4}}^{\frac{P}{2}} \left[ \left( -\frac{2}{p}x+1 \right) \cdot \sin \frac{2n\pi}{p} x \cdot dx \right] = \frac{4}{\pi^2 n^2} \cdot \sin \frac{n\pi}{2};
$$

But: 
$$
\sin \frac{n\pi}{2} = \begin{cases} 0, & n = 4k, 4k + 2 \\ 1, & n = 4k + 1 \\ -1, & n = 4k + 3 \end{cases}
$$
;

$$
f(x) = \sum_{n=1}^{\infty} \frac{4}{n^2} \cdot \sin \frac{n\pi}{2} \cdot \sin \frac{2n\pi}{2} x = \frac{4}{n^2} \cdot \sin \frac{2\pi}{n} x - \frac{4}{9n^2} \cdot \sin \frac{6\pi}{n} x + \frac{4}{2^2} \cdot \sin \frac{10\pi}{n} x + \dots =
$$
  
= 
$$
\frac{4}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \cdot \sin \frac{2(2n+1)\pi}{p} x ;
$$
(30)

$$
\tau_{\text{B}}(x) = \frac{1}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)^2} \cdot \sin \frac{2(2n+1)\pi}{p} x \tag{31}
$$

$$
u_{\mathbf{A}}(x) = K_0 \cdot \varphi_i \cdot \tau_0 \cdot \frac{R_1}{R_2} \cdot \frac{8}{\pi^2} \Sigma \dots + U_r
$$
 (32)

# $U<sub>0</sub>$

$$
u_{\mathbf{A}}(x) = U_{r} + U_{0} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^{2}} \cdot \cos \frac{2(2n+1)\pi}{p} x
$$
 (33)

also:

$$
u_{B}(x) = U_{r} + U_{0} \sum_{n=0}^{\infty} \frac{(-1)^{n}}{(2n+1)^{2}} \cdot \sin \frac{2(2n+1)\pi}{p} x
$$
 (34)

### **2. SUMMARY OF TECHNICAL-FUNCTIONAL CHARACTERISTICS**

 Resolution: 0,010 mm - without electronic subdivision and 0,001/0,0005 mm with electrical block on subdivision of 10 / 20

- $\checkmark$  Measurement interval: 10, 30, 50, 80, 100 mm
- Accuracy error max  $\pm$  1µm for 10, 30 mm and max  $\pm$  2 µm for 50, 80, 100 mm
- $\checkmark$  Maximum speed of the probe :0,4 m/s
- $\checkmark$  Lamp supply: +(5 ± 0,25)V
- Display capacity 8 decades + 1 sign decade
- ► Supply for electronic system  $_{220V_{ca} 10\%}^{+15\%}$  50 Hz ± 2%
- $\checkmark$  Supply: max 10 VA

### **3. CONCLUSIONS**

Mechatronic system with incremental photoelectric (linear) sensor is the second generation of opto-electronic displacement transducers, superior in terms of resolution and measurement accuracy.

Counting error is much smaller than the length of the measuring range, and their use does not require complicated numerical scheme with an analog converter that is a source of additional errors. Converting analog input size (linear mouvements) provides in out-put also a analog size / dimension (number of electrical oscillations) slightly to be digitized with comparator circuit.

Mechatronic system with high precision photoelectric incremental sensor for mini-dimensional measuring is used with specialized electronic equipment containing amplifier circuitry (90, floor trainer (10), which performs analog signal subdivision 5, floor divider (11) that the digital divide by 2 or 4 counting-floor display (12). Electronic block introduces the RESET, PRESET and the possibility of its use to control an external display.

### **REFERENCES**

- [1] G. Gheorghe "Ingineria Instrumentației"
- [2] Branebjerg, J. et. al. "A micromachined flow sensor for measuring small liquid flows", Proc. of IEEE Int. Conf. on Transducers, San Francisco, 1991, pp 41-46
- [3] Brand, S. Laux, T. und Tonshoff, H. "PiezoelektrischeAktoren" Mikroelektronik, 1992 (6) Fachbeilage "Mikroperipherik"
- [4] G. Gheorghe, L. Badita, A. Cirstoiu, S. Istriteanu, V. Despa and S. Ganatsios, "Mechatronics Galaxy", a New Concept for Developing Education in Engineering," Applied Mechanics and Materials, vol. 371, pp. 754-758, 2013.
- [5] G. Gheorghe, Tehnica măsurării inteligente, Bucureşti: CEFIN, 2010.
- [6] www.incdmtm.ro