THERMAL TRANSFER CONSIDERATIONS IN LAMINATED COMPOSITE PLATES. THERMAL FLUX AND INTERIOR TEMPERATURE WITH KNOWN VALUES

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Abstract. The paper specifies the temperatures developed between the layers of a composite, subjected to a thermal flux and interior temperature of the wall (plane, tubular, or spherical) with known values. It is envisaged an intimate or defective contact between the component layers. The transfer is considered manifested by convection and conduction. The established expressions can harness set in the case of other laminated structures, too, with materials of different natures, supported as needed.

Keywords: Laminated composite, thermal flux

1. INTRODUCTION

The actuality in the used materials domain for the industrial equipment achievement of all technical areas of activity often attract more attention to the stringent finding of some performing substitutes. A group that successfully enrolls in this competition is the laminated composites or plated figures. Not be neglected the continuing desire to increase the bearing capacities of the structures, while reducing their masses. In the practical exploitation of some mentioned structures, the thermal processes occur, that can develop important states of solicitation. It is necessary, in this regard, the temperatures knowledge that develop in the stratified walls privacy, essentially between the constituent layers in intimate contact or with native imperfections, respectively created in the work process [1-3].

This paper addresses, in this context, to the thermal transfer developed in plane, tubular or spherical walls, by convection. In addition to the main walls, the existence of some protections or some accidental deposits, which may adversely affect the thermal fields, is introduced in the analysis. The methodology of analysis, in this case, takes into account the knowledge of the interior temperature and the corresponding thermal flux, in which case the intermediate temperatures can be evaluated.

2. TYPES OF LAMINATED PLATES

2. 2. Plane plate

2.2. 1. Monolayer plane plate

Knowing the thermal flux from the interior surface of the plate, it is easy to deduce the temperature at the exterior surface of the wall, in the form [3]:

$$T_{e} = T_{i} - q_{p} \cdot \left(\delta / \lambda \right). \tag{1}$$

If there are any deposits and/or existence of some protections, the expressions of the intermediate temperatures have the characteristic equalities (by preserving the continuity of the thermal flux):

$$T_{d p i} = T_{i} - q_{p p} \left(\delta_{d i} / \lambda_{d i} \right) ; \qquad (2)$$

$$T_{1} = T_{i} - q_{pp} \left[\left(\delta_{di} / \lambda_{di} \right) + \left(\delta_{pi} / \lambda_{pi} \right) \right]; \quad (3)$$

$$T_{2} = T_{i} - q_{p} \left[\left(\delta_{di} / \lambda_{di} \right) + \left(\delta_{pi} / \lambda_{pi} \right) + \left(\delta / \lambda \right) \right];$$

$$(4)$$

$$T_{d p e} = T_{i} - q_{p p} \cdot \left[\begin{pmatrix} \delta_{d i} / \lambda_{d i} \end{pmatrix} + \begin{pmatrix} \delta_{p i} / \lambda_{p i} \end{pmatrix} + \\ + \begin{pmatrix} \delta / \lambda \end{pmatrix} + \begin{pmatrix} \delta_{p e} / \lambda_{p e} \end{pmatrix} \right]; \quad (5)$$

$$T_{e} = T_{i} - q_{pp} \cdot \left[\frac{\left(\delta_{di} / \lambda_{di} \right) + \left(\delta_{pi} / \lambda_{pi} \right) + \left(\delta_{de} / \lambda_{de} \right) + \left(\delta / \lambda \right) + \left(\delta_{pe} / \lambda_{pe} \right) + \left(\delta_{de} / \lambda_{de} \right) \right].$$
(6)

2. 2. 2. Multilayer plane plate, with intimate contact between the intermediate surfaces

This time, the following expressions for the temperature setting between layers is reached [3]:

$$T_{1,2} = T_{i} - q_{pp} \left(\delta_{1} / \lambda_{1} \right);$$
(7)

$$T_{2,3} = T_{i} - q_{pp} \left[\left(\delta_{1} / \lambda_{1} \right) + \left(\delta_{2} / \lambda_{2} \right) \right]; \quad (8)$$

$$T_{j, j+1} = T_{i} - q_{pp} \cdot \sum_{j=1}^{n-2} \left(\delta_{j} / \lambda_{j} \right); \qquad (9)$$

$$T_{n-1,n} = T_{i} - q_{pp} \sum_{j=1}^{n-1} \left(\delta_{j} / \lambda_{j} \right); \qquad (10)$$

$$T_{2} = T_{i} - q_{p} \cdot \sum_{j=1}^{n} \left(\delta_{j} / \lambda_{j} \right) = T_{i} - q_{p} \cdot \Re_{n}.$$
(11)

2. 2. 3. Multilayer plane plate, with intimate contact between the intermediate areas and/or protections and deposits

It is considered at the interior surface of the deposited layer (**if there are**, with the δ_{di} thickness and the λ_{di} thermal transfer coefficient) the T_i temperature or at the interior surface of the protection (**if there are**, with the δ_{pi} thickness and the λ_{pi} thermal transfer coefficient) respectively the T_e temperature on the exterior of the protection (**if there are**, with the δ_{pe} thickness and the λ_{pe} heat transfer coefficient), or at the exterior of the deposits(**if there are**, with the δ_{de} thickness and the λ_{de} thermal transfer coefficient).

Therefore, in the case of the existence both the protections and the deposits, the developed temperatures between layers are presented in the forms [3]:

$$T_{dpi} = T_{i} - q_{pp} \left(\delta_{di} / \lambda_{di} \right); \qquad (12)$$

$$T_{1} = T_{i} - q_{p} \left[\left(\delta_{di} / \lambda_{di} \right) + \left(\delta_{pi} / \lambda_{pi} \right) \right]; (13)$$

$$T_{1,2} = T_{i} - q_{p} \left[\left(\delta_{di} / \lambda_{di} \right) + \left(\delta_{pi} / \lambda_{pi} \right) + \left(\delta_{1} / \lambda_{1} \right) \right];$$
(14)

$$T_{n-1,n} = T_{i} - q_{p} \cdot \left[\begin{pmatrix} \delta_{di} / \lambda_{di} \end{pmatrix} + \begin{pmatrix} \delta_{pi} / \lambda_{pi} \end{pmatrix} + \\ + \sum_{j=1}^{n-1} \begin{pmatrix} \delta_{j} / \lambda_{j} \end{pmatrix} \right]; (16)$$

$$T_{2} = T_{i} - q_{pp} \cdot \left[\begin{pmatrix} \delta_{di} / \lambda_{di} \end{pmatrix} + \begin{pmatrix} \delta_{pi} / \lambda_{ii} \end{pmatrix} + \\ + \sum_{j=1}^{n} \begin{pmatrix} \delta_{j} / \lambda_{j} \end{pmatrix} \right]; \quad (17)$$

$$T_{d p e} = T_{i} - q_{p p} \cdot \left[\left(\delta_{d i} / \lambda_{d i} \right) + \left(\delta_{p i} / \lambda_{p i} \right) + \right] + \sum_{j=1}^{n} \left(\delta_{j} / \lambda_{j} \right) + \left(\delta_{p e} / \lambda_{p e} \right) \right];$$
(18)

$$T_{e} = T_{i} - q_{p} \cdot \left[\begin{pmatrix} \delta_{di} / \lambda_{di} \end{pmatrix} + \begin{pmatrix} \delta_{pi} / \lambda_{pi} \end{pmatrix} + \\ \begin{pmatrix} \delta_{pe} / \lambda_{pe} \end{pmatrix} + \begin{pmatrix} \delta_{de} / \lambda_{de} \end{pmatrix} + \\ + \sum_{j=1}^{n} \begin{pmatrix} \delta_{j} / \lambda_{j} \end{pmatrix} \right] =$$
(19)
$$= T_{i} - q_{pp} \cdot \Re_{p}^{\bullet},$$

with \mathfrak{R}_n^{\bullet} noting the total thermal resistance of the wall, including the protections and the deposits (possible).

2. 2. 4. Multilayer plane wall, with imperfect contact/defects between the intermediate areas and protections and/or deposits

In the industrial practice there are cases where between layers are created in advance, some "controlled imperfections" or some which are produced during the structure exploitation. In this way, it is necessary to analyze the conditions that the intermediate temperatures are produced. In the relationships derived in the following, the thermal resistances of these imperfections can be selected and accepted or rejected. We take into account the specified temperatures from the preceding paragraph, respectively T_i and T_e , in the established expressions, the intermediate thermal resistance been inserting. Thus, the (12) expression of the $T_{d,p,i}$ temperature is maintained, for the others intermediate temperatures it will establish:

$$T_{1} = T_{i} - q_{pp} \cdot \left[\begin{pmatrix} \delta_{di} / \lambda_{di} \end{pmatrix} + \begin{pmatrix} \delta_{pi} / \lambda_{pi} \end{pmatrix} + \\ + k_{dpi} \cdot \Re_{dpi} \end{pmatrix} ; \quad (20)$$

$$T_{1,2} = T_{i} - q_{pp} \cdot \left(\frac{\delta_{di}}{\lambda_{di}} + \frac{\delta_{pi}}{\lambda_{pi}} + \frac{\delta_{1}}{\lambda_{1}} + k_{dp} \cdot \Re_{dpi} + k_{p1} \cdot \Re_{p1} \right); (21)$$

$$T_{j, j+1} = T_{1} - q_{pp} \left\{ \frac{\delta_{di}}{\lambda_{di}} + \frac{\delta_{pi}}{\lambda_{pi}} + \sum_{j=1}^{n-1} k_{pj} \Re_{pj} + + \sum_{j=1}^{n-1} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \Re_{dpi} \right\};$$

$$j \in \left\{ 1, \cdots, (n-1) \right\}; \qquad (22)$$

$$T_{(n-1),n} = T_{1} - q_{pp} \cdot \left(\frac{\delta_{di}}{\lambda_{di}} + \frac{\delta_{pi}}{\lambda_{pi}} + \sum_{j=1}^{n-1} k_{pj} \cdot \Re_{pj} + + \sum_{j=1}^{n-1} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{dpi} \right);$$

$$T_{2} = T_{1} - q_{pp} \cdot \left(\frac{\delta_{di}}{\lambda_{di}} + \frac{\delta_{pi}}{\lambda_{pi}} + \sum_{j=1}^{n} k_{pj} \cdot \Re_{pj} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{dpi} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{dpi} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{dpi} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{pj} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + k_{dpi} \cdot \Re_{pj} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + \sum_{j=1}^{n} k_{pj} \cdot \Re_{pj} \right);$$

$$T_{e} = T_{1} - q_{pp} \cdot \left(\frac{\delta_{di}}{\lambda_{di}} + \frac{\delta_{pi}}{\lambda_{pi}} + k_{dpi} \cdot \Re_{dpi} + + k_{npe} \cdot \Re_{npe} + \frac{\delta_{pe}}{\lambda_{pe}} + + \frac{\delta_{de}}{\lambda_{de}} + k_{pde} \cdot \Re_{pde} + + \frac{\delta_{de}}{\lambda_{de}} + k_{pde} \cdot \Re_{pde} + + \frac{\delta_{de}}{\lambda_{de}} + k_{pde} \cdot \Re_{pde} + + \sum_{j=1}^{n} \frac{\delta_{j}}{\lambda_{j}} + \sum_{j=1}^{n} k_{pj} \cdot \Re_{pj} \right).$$

$$(26)$$

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In the previous equalities it has been noted: T_{dpi} , \Re_{dpi} – the temperature at the surface between the deposited and the protection layer from the interior of the wall, respectively the thermal resistance of the area with possible imperfections between the two layers; \Re_{pj} – the thermal resistance of the imperfect area located inside the *j* layer; T_{pde} , \Re_{pde} – the temperature and the thermal resistance of the imperfect area between the protection layer of its exterior and the deposited layer; \Re_{npe} – the thermal resistance of the imperfect area between the *n* layer of the wall and that of external protection.

<u>Note</u>: The thermal resistances of the areas with imperfections can be determined with expressions of the form:

$$\Re_{(j-1)j} = \left\{ \left[h_{r, e, (j-1)} + h_{r, i, j} \right] / \lambda_{(j-1), j} \right\}, (27)$$

with the corresponding notation: $h_{r, e, (j-1)}$ - the height of the roughness of the exterior surface of the (j-1) layer; $h_{r, i, j}$ - the height of the roughness of the interior surface of the j; layer; $\lambda_{(j-1), j}$ - the coefficient of the thermal transfer by conduction, that characterize the area with defects, which is considered by equalization with a composite containing a solid part (the roughness volume) and a gas or liquid phase (its volume), after case.

2.3. Tubular/cylindrical wall

2. 3. 1. Monolayer tubular/cylindrical wall

In this case, the expression for calculating the temperature from the exterior of the tubular wall is inferred [3]:

$$T_{e} = T_{i} - q_{pc} \left[\ln \left(d_{e} / d_{i} \right) / \left(2 \cdot \pi \cdot \lambda \right) \right].$$
(28)

Accepting the case, too, where protections and additional deposits (of granular solid materials or very viscous suspensions) in operation, are present, the $\Re_{T,5}$ total thermal resistance has, in this case, too, the expression specified in [3]. The intermediate temperatures between layers have the forms:

$$T_{d p i} = T_{i} - q_{p c} \cdot \left\{ \ln \left(d_{e d i} / d_{i d i} \right) / \left(2 \cdot \pi \cdot \lambda_{d i} \right) \right\};$$
(29)

$$T_{1} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\ln\left(d_{ede}/d_{idi}\right)}{\lambda_{di}} + \frac{\ln\left(d_{i}/d_{ipi}\right)}{\lambda_{pi}} \right)$$
(30)

$$T_{2} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{e} / d_{i} \right)}{\lambda} \right);$$

$$T_{pde} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} + \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{ipi} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{epe} / d_{e} \right)}{\lambda_{pe}} \right);$$
(31)

(32)

$$T_{e} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\ln\left(d_{edi}/d_{idi}\right)}{\lambda_{di}} + \frac{\ln\left(d_{i}/d_{ipi}\right)}{\lambda_{pi}} + \frac{\ln\left(d_{e}/d_{i}\right)}{\lambda_{pi}} + \frac{\ln\left(d_{epe}/d_{e}\right)}{\lambda_{pe}} + \frac{\ln\left(d_{ede}/d_{ide}\right)}{\lambda_{de}} + \frac{\ln\left(d_{ede}/d_{ide}\right)}{\lambda_{de}} \right)$$

$$(33)$$

In the previous equalities the notations were used: d_{edi} , d_{idi} – the exterior, respectively the interior diameter, of the interior deposits; d_{epi} , d_{ipi} – the exterior, respectively the interior diameter, of the interior protection, respectively of the protection; λ_{di} , λ_{pi} – the thermal conductivity coefficient of the interior deposits, respectively of the interior protection; λ_{de} , λ_{pe} – the thermal conductivity coefficient of the exterior deposits, respectively of the exterior protection. **Note**: If there is an intimate contact between layers:

$$d_{edi} \equiv d_{ipi}; d_{epi} \equiv d_{i};$$

$$d_{ipe} \equiv d_{e}; d_{epe} \equiv d_{ide}.$$
(34)

2. 3. 2. Multilayer tubular / cylindrical wall, without imperfections between layers

Following the previous methodology, we reach to the following expressions for the temperatures between the layers of the wall (for the $T_{d p i}$ and T_1 temperature, the equalities specified in [3] are kept):

$$T_{12} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{1}}} \right);$$

$$T_{23} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} + \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{e2} / d_{i2} \right)}{\lambda_{2}}}{\lambda_{2}} \right);$$
(35)
(36)

$$T_{j(j+1)} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} + \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} + \sum_{j=1}^{n-1} \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} \right);$$

$$j \in \left\{ 1, (n-1) \right\}; \qquad (37)$$

$$T_{2} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} + \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} + \sum_{j=1}^{n} \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} \right); \quad (38)$$

$$T_{pde} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}} +}{\frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}}} +}{\frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{pe}}} \right); \quad (39)$$

$$T_{e} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}}} +}{\frac{\ln \left(d_{epe} / d_{ipi} \right)}{\lambda_{pi}}} +}{\frac{\ln \left(d_{epe} / d_{ipi} \right)}{\lambda_{pi}}} +} \right) - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\sum_{j=1}^{n} \frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{dj}} +}{\frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{pi}}} +}{\frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{pe}}} +} \right), \quad (40)$$

 $+\frac{\ln\left(d_{e d e}/d_{i d e}\right)}{\lambda_{d e}}$

noting with d_{ej} , d_{ij} – the exterior, respectively the interior diameter of the j; layer; λ_j – the coefficient of the thermal conductivity of the j; layer.

2. 3. 3. Multilayered tubular/cylindrical wall, with imperfections between layers

As in the previous cases, when defects occur between layers, or inserted zones from the initial phase, with controlled geometry, the temperatures expressions between layers are changed accordingly:

$$T_{1} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln\left(d_{ede}/d_{idi}\right)}{\lambda_{di}} +}{\frac{\ln\left(d_{i}/d_{ipi}\right)}{\lambda_{pi}} + k_{dpi} \cdot \Re_{dpi}} \right);$$

$$(41)$$

$$T_{12} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}}} + \frac{\frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{pi}} +}{\frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{1}}} + \frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{1}} + \frac{k_{dpi} \cdot \Re_{dpi} + k_{p1} \cdot \Re_{p1}}{\lambda_{1}} \right); \quad (42)$$

$$T_{23} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{+ \frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}}} + \frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{1}} + k_{dpi} \cdot \Re_{dpi} + \frac{\ln \left(d_{e1} / d_{i1} \right)}{\lambda_{1}} + k_{p1} \cdot \Re_{p1} + k_{p2} \cdot \Re_{p2}} \right);$$
(43)

$$T_{j(j+1)} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\begin{array}{c} \frac{\ln\left(d_{edi}/d_{idi}\right)}{\lambda_{di}} + \\ + \frac{\ln\left(d_{i}/d_{ipi}\right)}{\lambda_{pi}} + \\ + \sum_{j=1}^{n-1} k_{pj} \cdot \Re_{pj} \\ + \sum_{j=1}^{n-1} \frac{\ln\left(d_{ej}/d_{ij}\right)}{\lambda_{dj}} + \\ + k_{dpi} \cdot \Re_{dpi} + \\ \end{array} \right);$$

$$j \in \left\{ 1, (n-1) \right\}; \qquad (44)$$

$$T_{2} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}}} + \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} + \frac{\sum_{j=1}^{n} \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} +}{\frac{1}{2} + k_{pdi} \cdot \Re_{pdi}} + \sum_{j=1}^{n} k_{pj} \cdot \Re_{pj}} \right);$$
(45)

$$T_{pde} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\frac{\frac{\ln \left(d_{edi} / d_{idi} \right)}{\lambda_{di}} +}{\frac{\ln \left(d_{i} / d_{ipi} \right)}{\lambda_{pi}}} + \frac{\ln \left(d_{ej} / d_{ij} \right)}{\lambda_{dj}} + \frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{pe}} + \frac{\ln \left(d_{epe} / d_{ipe} \right)}{\lambda_{pe}} + \frac{k_{pdi} \cdot \Re_{pdi} + k_{npe} \cdot \Re_{npe}}{k_{pdi} \cdot \Re_{pj} \cdot \Re_{pj}} \right)$$

$$(46)$$

$$T_{e} = T_{i} - \frac{q_{pc}}{2 \cdot \pi} \cdot \left(\begin{array}{c} \frac{\ln\left(d_{edi}/d_{idi}\right)}{\lambda_{di}} + \\ + \frac{\ln\left(d_{i}/d_{ipi}\right)}{\lambda_{pi}} + \\ + \frac{\ln\left(d_{ej}/d_{ij}\right)}{\lambda_{dj}} + \\ + \frac{\ln\left(d_{epe}/d_{ipe}\right)}{\lambda_{pe}} + \\ + k_{pdi} \cdot \Re_{pdi} + k_{pde} \cdot \Re_{pde} + \\ + \sum_{j=1}^{n} k_{pj} \cdot \Re_{pj} + k_{npe} \cdot \Re_{npe} \end{array} \right)$$

$$(47)$$

In the previous equalities we were noted: \Re_{pj} – the thermal resistance of the imperfect area, located at the interior j; layer; \Re_{npe} – the thermal resistance of the imperfect layer, located at the exterior of the *n* layer of the wall (between it and the exterior protection). It is noted that the temperatures values between layers, in this context, diminishes because the influence of the imperfections, or of the areas with adequate geometry to this purpose.

2. 4 Spherical wall

2. 4. 1. Monolayer spherical wall

Knowing the values of the interior temperature and of the thermal flux, the temperature at its exterior, evaluated with the relationship, result:

$$T_{e} = T_{i} - \frac{q_{ps}}{2 \cdot \pi \cdot \lambda} \cdot \frac{d_{e} - d_{i}}{d_{i} \cdot d_{e}}.$$
(48)

If it is considered, in this case, deposits and protections on the interior and exterior surface of the wall are present, the intermediate temperatures have the expressions:

$$T_{d p i} = T_{i} - \frac{q_{p s}}{2 \cdot \pi \cdot \lambda_{d i}} \cdot \frac{d_{e d i} - d_{i d i}}{d_{e d i} \cdot d_{i d i}}; \quad (49)$$

$$T_{1} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} \right); \quad (50)$$

$$T_{2} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}} + \frac{\frac{d_{e} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}}{+ \frac{d_{e} - d_{i}}{\lambda \cdot d_{i} \cdot d_{e}}} + \right);$$
(51)

$$T_{pde} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}} + \frac{\frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{e}}}{+ \frac{d_{epe} - d_{i}}{\lambda \cdot d_{i} \cdot d_{e}}} + \frac{\frac{d_{epe} - d_{ipe}}{\lambda_{pi} \cdot d_{ipe} \cdot d_{epe}}}{+ \frac{d_{epe} - d_{ipe}}{\lambda_{pi} \cdot d_{ipe} \cdot d_{epe}}} \right);$$
(52)

$$T_{e} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \frac{d_{e} - d_{i}}{\lambda \cdot d_{i} \cdot d_{e}} + \frac{\frac{d_{epe} - d_{ipe}}{\lambda_{pe} \cdot d_{ipe} \cdot d_{epe}}}{+ \frac{d_{ede} - d_{ide}}{\lambda_{de} \cdot d_{ide} \cdot d_{ede}}} + \frac{\frac{d_{ede} - d_{ide}}{\lambda_{de} \cdot d_{ide} \cdot d_{ede}}}{- \frac{d_{ide}}{\lambda_{de} \cdot d_{ide} \cdot d_{ede}}} \right).$$
(53)

In the previous equalities the notations were used: d_{edi} , d_{idi} – the exterior, respectively the interior diameter, of the interior deposits; d_{epi} , d_{ipi} – the exterior, respectively the interior diameter, of the interior protection, respectively of the protection; λ_{di} , λ_{pi} – the thermal conductivity coefficient of the interior deposits, respectively of the interior protection; λ_{de} , λ_{pe} – the thermal conductivity coefficient of the exterior deposit, respectively of the exterior protection.

2. 4. 2. Multilayered spherical wall, without imperfections between layers and without protections and/or deposits

The equalities for calculating the intermediate temperatures have the forms (the given expressions are kept for $T_{d,p,i}$ and T_{\perp}) [3]:

$$T_{12} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di'} d_{idi'} d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi'} d_{ipi'} d_{epi}}}{+ \frac{d_{e1} - d_{i1}}{\lambda_{1} \cdot d_{i1'} d_{e1}}} + \right); \quad (54)$$

$$T_{23} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di'} d_{idi'} d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi'} d_{ipi'} d_{epi}}} + \frac{d_{e2} - d_{i2}}{\lambda_{2} \cdot d_{i2'} d_{i2'} d_{e2}} \right); \quad (55)$$

$$T_{j(j+1)} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}} + \frac{\frac{d_{ej} - d_{ij}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}}}{+ \sum_{j=1}^{n-1} \frac{d_{ej} - d_{ij}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}}} \right);$$

$$j \in \left\{ 1, (n-1) \right\}; \qquad (56)$$

$$T_{n(n+1)} = T_{2} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{+ \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}} + \frac{1}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \frac{1}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \frac{1}{\lambda_{pi} \cdot d_{ij} \cdot d_{ej}} \right);$$

$$(-d_{pi} = d_{pi} - d_{pi})$$
(57)

$$T_{p d e} = T_{i} - \frac{q_{p s}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{e d i} - d_{i d i}}{\lambda_{d i} \cdot d_{i d i} \cdot d_{e d e}}}{+ \frac{d_{e p i} - d_{i p i}}{\lambda_{p i} \cdot d_{i p i} \cdot d_{e p i}}} + \frac{\frac{d_{e p e} - d_{i p i}}{\lambda_{p i} \cdot d_{i p i} \cdot d_{e p i}}}{+ \frac{\lambda_{p e} - d_{i p}}{\lambda_{p e} \cdot d_{i p e} \cdot d_{e p e}}} + \frac{\frac{d_{e p e} - d_{i p}}{\lambda_{p e} \cdot d_{i p e} \cdot d_{e p e}}}{+ \frac{d_{e p e} - d_{i p e}}{\lambda_{p e} \cdot d_{i p e} \cdot d_{e p e}}} \right); \quad (58)$$

$$T_{e} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}}}{\frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}}} + \frac{\frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ij} \cdot d_{ej}}}{\frac{\lambda_{j} \cdot d_{ij} \cdot d_{ej}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}}} + \frac{\frac{d_{epe} - d_{ipe}}{\lambda_{pe} \cdot d_{ipe} \cdot d_{epe}}}{\frac{\lambda_{pe} \cdot d_{ipe} \cdot d_{epe}}{\lambda_{de} \cdot d_{ide} \cdot d_{ede}}} \right).$$
(59)

2. 4. 3. Multilayered spherical walls, with imperfections between layers and with protections and/or deposits

It takes up the mentioned idea, when imperfections can intervene in the functioning of the spherical wall or, why not, some areas of production stage with controlled geometry, in order to thermal or sound isolate. As a result, the relationships for the intermediate temperatures adjust, as follows:

$$T_{1} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} +}{\frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + k_{dpi} \cdot \Re_{dpi}} \right);$$
(60)

$$T_{12} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \begin{pmatrix} \frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + \\ + \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \\ + \frac{d_{e1} - d_{i1}}{\lambda_{1} \cdot d_{i1} \cdot d_{e1}} + \\ + k_{dpi} \cdot \Re_{dpi} + k_{p1} \cdot \Re_{p1} \end{pmatrix}; (61)$$

$$T_{23} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + k_{dpi} \cdot \Re_{dpi} + \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + k_{p1} \cdot \Re_{p1} + \frac{d_{e1} - d_{i1}}{\lambda_{1} \cdot d_{i1} \cdot d_{e1}} + k_{p2} \cdot \Re_{p2} \right);$$
(62)

$$T_{j(j+1)} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\begin{array}{c} \frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + \\ + \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \\ + \frac{\sum_{j=1}^{n-1} \frac{d_{ej} - d_{ij}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}}} + \\ + \sum_{j=1}^{n-1} \frac{d_{ej} - d_{ij}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}} + \\ + k_{dpi} \cdot \Re_{dpi} + \sum_{j=1}^{n-1} k_{pj} \cdot \Re_{pj} \right); \\ j \in \left\{ 1, (n-1) \right\}; \quad (63)$$

$$T_{n(n+1)} = T_{2} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \cdot \left(\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + \frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + \frac{d_{ej} - d_{ij}}{\lambda_{ji} \cdot d_{ij} \cdot d_{ej}} + \frac{\sum_{j=1}^{n} \frac{d_{ej} - d_{ij}}{\lambda_{ji} \cdot d_{ij} \cdot d_{ej}}}{\sum_{j=1}^{n} \frac{d_{ej} - d_{ij}}{\lambda_{ji} \cdot d_{ij} \cdot d_{ej}}} \right)$$

$$-\frac{q_{ps}}{2\cdot\pi}\cdot\begin{pmatrix}+k_{dpi}\cdot\mathfrak{R}_{dpi}+\\+\sum_{j=1}^{n}k_{pj}\cdot\mathfrak{R}_{pj}\end{pmatrix};$$
(64)

$$T_{p d e} = T_{i} - \frac{q_{p s}}{2 \cdot \pi} \cdot \left(\frac{\frac{d_{e d i} - d_{i d i}}{\lambda_{i d i} d_{i d i} \cdot d_{e d e}} + k_{d p i} \cdot \Re_{d p i} + \frac{d_{e p i} - d_{i p i}}{\lambda_{p i} \cdot d_{i p i} \cdot d_{e p i}} + k_{n p e} \cdot \Re_{n p e} + \frac{\lambda_{p i} \cdot d_{i p i} \cdot d_{e p i}}{\lambda_{j} \cdot d_{i j} \cdot d_{e j}} + \frac{\sum_{j=1}^{n} \frac{d_{e j} - d_{i j}}{\lambda_{j} \cdot d_{i j} \cdot d_{e j}}}{\sum_{j=1}^{n} k_{p j} \cdot \Re_{p j}} \right);$$
(65)

$$T_{e} = T_{i} - \frac{q_{ps}}{2 \cdot \pi} \left(\frac{\frac{d_{edi} - d_{idi}}{\lambda_{di} \cdot d_{idi} \cdot d_{ede}} + k_{npe} \cdot \Re_{npe} + \frac{\frac{d_{epi} - d_{ipi}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + k_{pde} \cdot \Re_{pde}}{\lambda_{pi} \cdot d_{ipi} \cdot d_{epi}} + k_{dpi} \cdot \Re_{dpi} + \frac{\sum_{j=1}^{n} \frac{d_{ej} - d_{ij}}{\lambda_{j} \cdot d_{ij} \cdot d_{ej}} + k_{dpi} \cdot \Re_{dpi}}{\sum_{j=1}^{n} k_{pj} \cdot \Re_{pj}} \right)$$
(66)

Important note: If the case when the transfer by convection is present, the expressions set for the intermediate temperatures affect appropriate, taking into account the specifications from the corresponding sections. Therefore, the total thermal resistance increases, which adversely affects the intermediate temperatures.

3. CONCLUSIONS

This paper addresses the problem of the thermal transfer by laminated plates, taking into account both the increase of the thermal flux and of the temperature of the interior of the plate/wall, on the one hand, and the thermal characteristics and their geometry, on the other hand.

The present study assesses the evaluation of the intermediate temperatures between layers. The mathematical expressions are derived in the case of laminated structures, in intimate contact with each other, and in the case of functional defects are present, or depositing of highly viscous or solid products.

The attention is to the role of the imperfections, which can be thought, too, by the presence of some areas with controlled geometry, with a similar function, but may favor the thermal or sound isolation of the structure.

The calculation formulas can be applied both for composite laminates, as well as in other practical cases (plating, combinations of structures with metallic and non-metallic/inorganic material, for example, as well as ribbed plates).

4. REFERENCES

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