MATHEMATICAL SOLUTION FOR THE SOLDIFICATION PROCESS MODELLING IN STEEL CONTINUOUS CASTING, CONSDERING THE CONVECTION AT THE LIQUID-SOLID INTERFACE

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Abstract *This study proposes a strictly mathematical solution for the mathematical model which considers the convection at the liquid-solid interface in the steel continuous casting solidification process. The obtained solutions consist of calculation relations of: the thickness of the solidified crust, the temperature distribution in the solid fraction and the convection ratio at the liquid-solid interface.*

Keywords: mathematical model, solidification process, continuous casting of steel

1. INTRODUCTION

In order to emphasize the particularities of the continuous casting process we can use the mathematical modeling, able to give answers to any change of the working conditions. The mathematical model by active experiment, under dynamic conditions and on heat model, can reproduce the studied process through fundamental relations and shortly allows establishing the optimum technological option with minimum expenses.

In the specialized literature, there have been proposed models of the steel solidification in the continuous casting which took into consideration the convection and conduction heat exchange, however insufficiently evaluating the influence of the convective thermal exchange on the evolution of the solidification line.
The proposed simplified assumptions neglected: the overheating and the convection in the liquid steel, the mass transfer as well as the heat exchange through convection on the casting direction. However:

-the convective movements may change the thermal state of the liquid alloy from the solidification cone, rendering uniform the temperature distribution and improving the inner structure of the continuously casted semi-products;

- a small thickness of the solidified crust at the meniscus level may curve the surface of the ¨yarn¨ towards the crystallizer wall, thus leading to an irregular surface of the semi-product.

- the solidification study, considering the convective phenomenon, leads to the right positioning of the variation in the solidification line and, implicitly, of the thickness of the solidified crust. The determination of the solidified crust thickness may lead to the casting speed increase keeping, in the same time, a high quality of the continuously casted semi-products.

Many authors proposed models which stay insufficient for assessing: the influence of the convective movements, the influence of the overheating on the thermal state of the system, temperature distribution in the solid-liquid interface, the position of the solidification interface (the thickness of the semi-product crust); the necessary evaluations for establishing an optimum running process n a continuous casting installation

2. MATHEMATICAL MODEL DEFINITION FOR THE SOLIDIFICATION PROCESS IN THE STEEL CONTINUOUS CASTING, CONSIDERING THE CONVECTION AT THE LIQUID-SOLID INTERFACE

Begun in the crystallizer and finished at the exit from the secondary cooling zone, the cooling of the continuously casted semi-product is the key to obtain a superior quality [4].

The cooling of the ¨yarn¨ should be made under control by following the temperature distribution and not only by following the quantity of water used for the cooling process. Thus we propose ourselves, by addressing the mathematical model, to be able to acertain: the temperature distribution in the solidified yarn; the thickness of the solidified crust and the ratio value of the convection at the liquid – solid interface.

The equations on which the mathematical model is based

The heat transfer is negligible on the casting direction and there is considered only on the direction of the normal on the surface of the crystallizer. Due to the specific of the squared profiles we address the study on only one direction and for only half of the semi-product. In the solidified alloy, the heat exchange takes place through conduction.
The specific Fourier equation of the system is:

$$\frac{\partial T_x}{\partial t} = a \frac{\partial^2 T_x}{\partial x^2} \tag{1}$$

where :

$a = \frac{\lambda_s}{\rho_s \cdot C_s}$ is the solidified alloy heat diffusivity;

λ_s – heat conductivity;

ρ_s - density;

C_s – specific heat;

T – the temperature of the considered point;
t - time;
x – the coordinate of the point.
Relation (1) stands for the range for which x complies with the condition: $0 \le x \le b-\xi$ (2)
where: ξ is the position of the solidification line.
The temperature at the initial moment t = 0 is uniform and equals the casting temperature (T_t – casting temperature). The solidification begins at the moment t = 0 with the yarn rind $b-\xi = 0$

$$T(0,\xi_0) = T_t \tag{3}$$

The temperature at the solidified crust stays constant in time and it is marked by T_c (this temperature is measured). At the temperature T_s the solidification of the piece occurs as a consequence of the crystallization latent heat L.

$$T_s \in (T_c, T_t) \tag{4}$$

The heat transfer within the liquid alloy is made through convection.
The heat transfer between the solidification line and the liquid metal occurs through convection and is associated to the solidification latent heat [6] and it will satisfy the equation:

$$-\lambda(T)\frac{\partial T}{\partial x} = L\rho\frac{d\zeta}{dt} + h\Delta T \tag{5}$$

In the solidification line, the temperature $T(t,\xi(t))$ will satisfy the condition:

$$T(t,\zeta(t)) = T_s \tag{6}$$

In the liquid metal the heat transfer occurs through convection, and the development of the liquid metal temperature is given by the equation:

$$(b-\zeta)\cdot\rho C\frac{\partial \Delta T}{\partial t} = -h\Delta T \tag{7}$$

where: b = ½ od the semi-product width
ΔT – the overheating.

3. ASSESSING THE MATHEMATICAL MODEL

The conditions, at the limit, of the issue are the followings:

The solidification begins at the moment t = 0 with the rind of the yarn:

- at t = 0; $T(0,\xi(0)) = T_t$ meaning, $T_x = T_t$ (8)
- at x = 0; $T_{(t,0)} = T_c$ = const. meaning, $T_x = T_c$ (9)
- at $x = b-\xi$; $T_{(t,\xi)} = T_s$ meaning, $T_x = T_s$ (10)

where:

T_t is the casting temperature;

T_s –the alloy temperature at solidification;

T_c – the surface temperature of the solidified yarn;

L – the solidification latent heat.

The solution of the differential equation (1) is as:

$$T_x = A + B\,\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right) \tag{11}$$

where: A, B are constants.

Constant A is calculated by condition (9).

at x = 0 we have erf (0) = 0, then $T_x = A$, therefore A $=T_C$

we replace A in relation (11) and then we obtain:

$$T_x = T_c + B\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right) \tag{12}$$

Constant B is calculated by condition (10):

$$T_x = T_c + B\mathrm{erf}\left(\frac{b-\zeta}{2\sqrt{at}}\right) = T_s \tag{13}$$

To determine constant B there s adopted :

$$b-\zeta = K_s\cdot\sqrt{t} \tag{14}$$

Deriving by t, obtwe obtain:

$$\frac{d(b-\zeta)}{dt} = \frac{K_s}{2\sqrt{t}} \tag{15}$$

From relation (13), by replacing $b-\xi$ from relation (14) constant B s calculated:

$$B = \frac{T_s - T_c}{\mathrm{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \tag{16}$$

Introducing constant B in (13) we obtain:

$$T_x = T_c + \frac{T_s - T_c}{\mathrm{erf}\left(\frac{K_s}{2\sqrt{a}}\right)}\cdot\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right) \tag{17}$$

By separating the variables relation (17) becomes:

$$\frac{T_x - T_c}{T_s - T_c} = \frac{\mathrm{erf}\left(\frac{x}{2\sqrt{at}}\right)}{\mathrm{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \tag{18}$$

By relation (18) we have the temperature distribution in the solidified alloy.

The partial derivative $\frac{\partial T_x}{\partial x}$ as per equation (17) will be:

$$\frac{\partial T_x}{\partial x} = \frac{T_s - T_c}{\operatorname{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \cdot \frac{\exp\left(-\frac{x^2}{4at}\right)}{\sqrt{\pi at}} \tag{19}$$

At the liquid-solid interface equation (5) will become:

$$-\lambda(T)\frac{T_s - T_c}{\operatorname{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \cdot \frac{\exp\left(-\frac{x^2}{4at}\right)}{\sqrt{\pi at}} = L \cdot \rho \cdot \frac{d\zeta}{dt} + h\Delta T \tag{20}$$

By replacing in relation (20) ξ from relation (14) we wll obtain:

$$-\lambda(T)\frac{T_s - T_c}{\operatorname{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \cdot \frac{\exp\left(-\frac{K_s^2}{4a}\right)}{\sqrt{\pi at}} =- L \cdot \rho \cdot \frac{K_s}{2\sqrt{t}} + h\Delta T \tag{21}$$

We adopt:

$$K_s = \sqrt{\frac{2\Delta T_s \lambda}{\rho \cdot L}}$$

$$\Delta T_s = T_s - T_c$$

From relation (21) we calculate ΔT and we derive it by t and we introduce it in relation (7) to obtain b-ξ.

$$\Delta T = \left(\frac{1}{h}\left[-\lambda(T)\frac{T_s - T_c}{\operatorname{erf}\left(\frac{K_s}{2\sqrt{a}}\right)}\frac{\exp\left(-\frac{K_s^2}{4a}\right)}{\sqrt{\pi a}} + L\rho\frac{K_s}{2}\right]\right)\frac{1}{\sqrt{t}} \tag{22}$$

$$\frac{\partial \Delta T}{\partial t} = \frac{1}{2}\cdot\frac{1}{h}\cdot\frac{1}{t\sqrt{t}}\left(\lambda(T)\frac{T_s - T_c}{\operatorname{erf}\left(\frac{K}{2\sqrt{a}}\right)}\frac{\exp\left(-\frac{K_s^2}{4a}\right)}{\sqrt{\pi a}} - L\rho\frac{K_s}{2}\right) \tag{23}$$

The expression of b-ξ will be:

$$b - \zeta = \frac{2ht}{\rho C} \tag{24}$$

$\Delta T = T_t - T_s$ by applying condition (8) we obtain the value for h:

$$h = \frac{1}{\Delta T}\left[L\rho\frac{K_s}{2\sqrt{t}} - \lambda(T)\frac{T_s - T_c}{\operatorname{erf}\left(\frac{K_s}{2\sqrt{a}}\right)} \cdot \frac{\exp\left(-\frac{K_s^2}{4a}\right)}{4\sqrt{\pi at}}\right] \tag{25}$$

The thermo-dynamic and thermo-physical quantities which depend on the temperature, at the solidification of the continuously casted steel may be calculated using the relations [3],[10]:

$$\rho_{(T)} = 7840\frac{1}{1+\alpha(T)\cdot(T-T_0)} \tag{26}$$

where:

$$\alpha_{(T)}10^6 = 10{,}7 + 0{,}6\left(\frac{T-273{,}16}{100}\right) - \frac{2{,}9}{\operatorname{ch}\left[0{,}76\left(\frac{T-T_0}{100}\right)^2\right]}$$

T_0=1178,16K

For T<1041K, $C_p(T)$ it will be calculated by the relation :

$$C_p(T) = 1000\left[0{,}4814 + 0{,}1997\left(\frac{T-273{,}16}{100}\right) + 0{,}812 \cdot e^{-0{,}0099(1041-T)}\right] \tag{27}$$

For T > 1041K, $C_p(T)$ it will be calculated by the relation:

$$C_p(T) = 1000\left[0{,}4814 + 0{,}1997\left(\frac{T-273{,}16}{100}\right) + 0{,}812 \cdot e^{-0{,}0261(1041-T)}\right] \tag{28}$$

For T ≤ 1173K, $\lambda(T)$ it will be calculated by the relation:

$$\lambda(T) = 26{,}74 + \frac{(\lambda_0 - 26{,}74)(1173{,}16 - T)}{1173} \tag{29}$$

where:

$$\lambda_0 = 74{,}42 - 16{,}28[\%C] - 34{,}88[\%Si] - 23{,}36[\%Mn], \quad (30)$$

For T > 1173K, λ(T) it will be calculated with the relation:

$$\lambda(T) = 0{,}01164 \cdot T + 13{,}08628 \quad (31)$$

For the calculation of the thermo-physical quantities which vary along with the temperature and are entry data the following relations can be used [3],[10]:

$$C_p = \left(23{,}57 + 9{,}75T \cdot 10^{-3}\right)4{,}826 \quad (32)$$

$$\rho_l = \left[10{,}678 - 13{,}17 \cdot 10^{-4}\left(T_l - T_s\right)\right] \cdot 10^3 \quad (33)$$

$$a = \frac{\lambda_s}{\rho_s \cdot C_{ps}} \quad (34)$$

$$K_s = \sqrt{\frac{2\Delta T_s \lambda_l}{\rho_l \cdot L}} \quad (35)$$

where: C_{pl} is the specific heat of the liquid;
C_{ps} - the specific heat of the solid;
ρ_l - density of the liquid;
ρ_s - density of the solid;
a - alloy thermal diffusivity in solid state;
K_s - solidification constant;
L - crystallization latent heat;
$\Delta T_s = T_s - T_c$
λ_s – thermal conductivity of the solid;
λ_l - thermal conductivity of the liquid;
T_l - the temperature of the liquid alloy.
T_s - solidification;
T_c - the solidified crust temperature.

4. CONCLUSIONS

By solving the mathematical model which considers the convection heat transfer at the liquid-solid interface the calculation relations were obtained for: the temperature distribution in the solid fraction, the thickness of the solidification crust and the convection ratio at the liquid-sold interface.

The obtained solutions clearly emphasize the influence of the convective movements in the solidification cone and of the convection at the liquid-solid interface, contributing in the same time to the explanation of the phenomena taking place at the crystallizer level in the solidification of the continuously casted semi-products.

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