ASPECTS OF THE PARAMETRIC ANALYSIS OF HYPERSTATIC PLANE SYSTEMS COMPOSED OF TWO WELDED BEAMS (I)

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Abstract: *The parametric analysis and design became of interest in the last time period due to the development of the computing technology and are widely used by the professional software. The parametric design is employed in the 2D and 3D professional modeling software, because it allows the reconstruction and change of the virtual model by changing one or more dimensional parameters. This work presents a simple example of parametric analysis of hyperstatic plane systems composed of two perpendicularly welded beams of the same length, using the force method for various loading scenarios with concentrated and distributed loads and bending moments.*

Keywords: *parametric analysis, hyperstatic (statically indeterminate) systems, force method.*

1. INTRODUCTION

A plane hyperstatic frame is considered, composed of two perpendicularly welded beams of the same length *L,* fixed at the both ends, subjected to a loading scenario represented in the Fig. 1. The beams have a constant cross-section and are subjected to bending, shearing and/or tension/compression under the action of:

- the concentrated forces P_1 and P_2 acting on the two beams at the locations $\alpha_P L$ and $\beta_P L$;
- the bending moments N_1 and N_2 acting on the two beams at the locations $\alpha_N L$ and $\beta_N L$;
- the uniform distributed loads q_1 and q_2 acting on the beam segments μL and θL respectively, at the locations ηL and ωL.

For the particular values of the parameters, given in the Table 1, the reaction forces in the supports A and B will be calculated, along with the rotation of the section C.

Table 1. Particular values of the parameters.

N_1	PL	α_N	1/4
N_2	-	β_N	-
P_1	-	α_P	-
P_2	$-P$	β_P	2/3
q_1	$-2P/L$	η	1/2
		μ	1/2
q_2	$-2P/L$	ω	0
		θ	1/2

The parametric analysis of hyperstatic frame is represented by the determination of the parametric expressions of displacements for the base system and by the numerical computations for different particular values of the parameters.

2. FORCE METHOD

The force method allows the determination of the hyperstatic unknowns by choosing a statically determined system (called base system), which is mechanically equivalent to the given hyperstatic system. The base system is obtained by replacing the fixed support in the point B with mechanical elements: the forces X_1 and X_2 and the moment X_3 – which are the hyperstatic unknowns (Fig. 2).

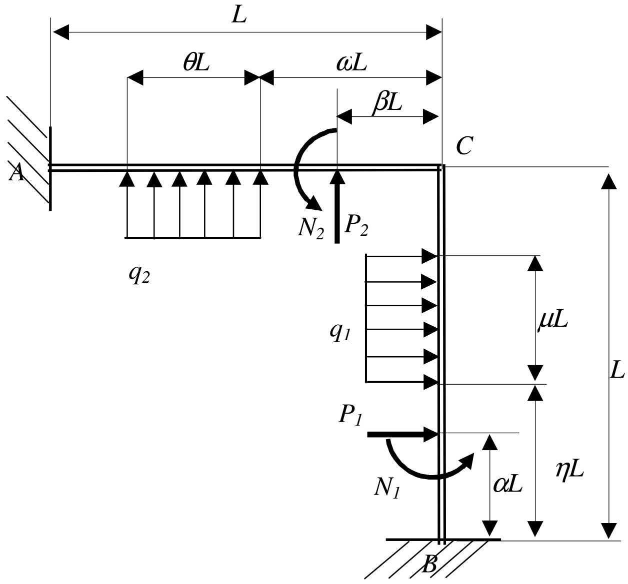

Fig. 1. Frame layout and loading scheme.

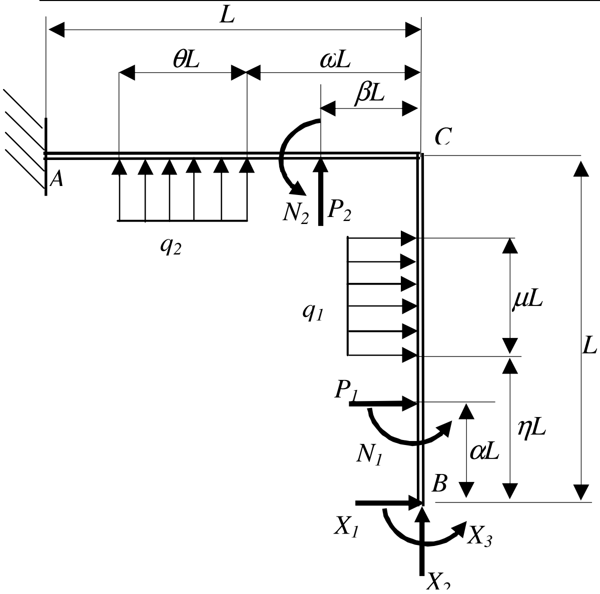

Fig. 2. The base system and the hyperstatic unknowns.

The canonical equations of the force method are:

$$\begin{cases} \delta_{10} + \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \delta_{13} \cdot X_3 = 0 \\ \delta_{20} + \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \delta_{23} \cdot X_3 = 0 \\ \delta_{30} + \delta_{31} \cdot X_1 + \delta_{32} \cdot X_2 + \delta_{33} \cdot X_3 = 0 \end{cases} \quad (1)$$

where:

δ_{10}, δ_{20} and δ_{30} are the displacements produced by the given forces in the base system, on the direction of the hyperstatic efforts X_1, X_2 and X_3;
δ_{ij} are the displacements produced by the unit foces $X_j = 1$, $j=1,2$ and unit bending moment $X_3 = 1$ in the base system on the direction X_i.

The determination of the displacements δ_{10}, δ_{20} and δ_{30} and δ_{11}, δ_{12}, … δ_{33} for the base system in Fig. 2 will be performed using the Mohr-Maxwell method, considering only the displacements produced by bending moments (Fig. 3, Fig. 4, Fig. 5):

$$\delta_{i0} = \sum \int \frac{m_i M^0}{EI} dx \; ; \qquad \delta_{ik} = \sum \int \frac{m_i m_k}{EI} dx \qquad i,k = 1,2,3. \quad (2)$$

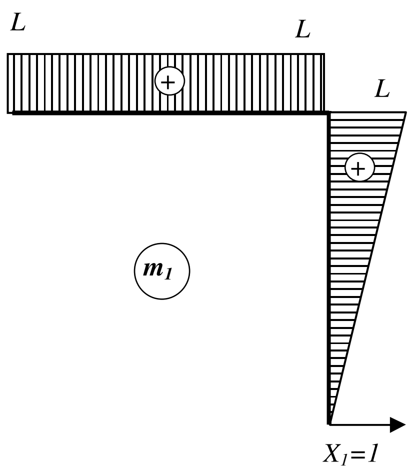

Fig. 3. The m₁ diagram.

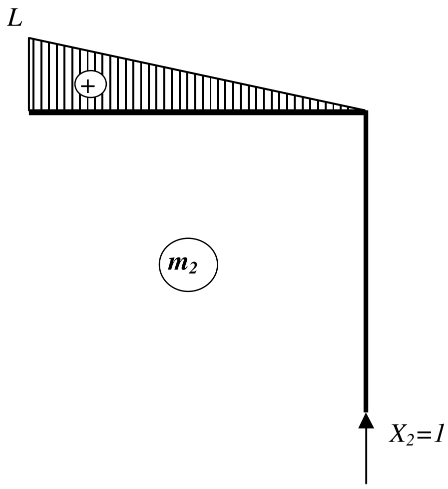

Fig. 4. The m₂ diagram.

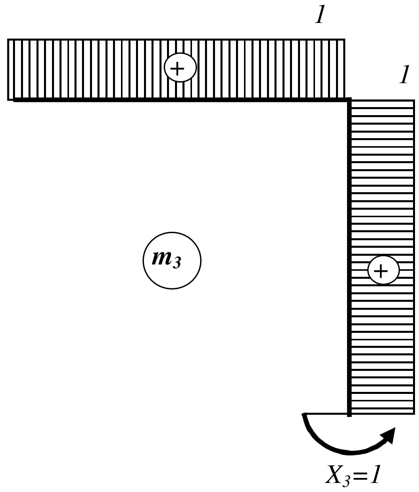

Fig. 5. The m₃ diagram.

3. OBTAINED RESULTS

The displacements corresponding to the unit forces δ_{ij} were determined using the equations (2) and the obtained values are presented in Table 2.

Table 2. Displacements corresponding to the unit forces.

$\delta_{11} = \frac{4}{3} \cdot \frac{L^3}{EI}$	$\delta_{12} = \delta_{21} = \frac{1}{2} \frac{L^3}{EI}$
$\delta_{13} = \delta_{31} = \frac{3}{2} \cdot \frac{L^2}{EI}$	$\delta_{22} = \frac{1}{3} \cdot \frac{L^3}{EI}$
$\delta_{23} = \delta_{32} = \frac{1}{2} \cdot \frac{L^2}{EI}$	$\delta_{33} = 2 \cdot \frac{L}{EI}$

The parametric expressions of the displacements δ_{10}, δ_{20} and δ_{30} as function of α, β, η, μ, ω and θ are obtained for each of the six loads, which independently act on the base system, using the diagrams in Fig. 36 - Fig. 11. The loads were considered as: $N_i = PL$; $P_i = P$; $q_{ij} = P/L$.

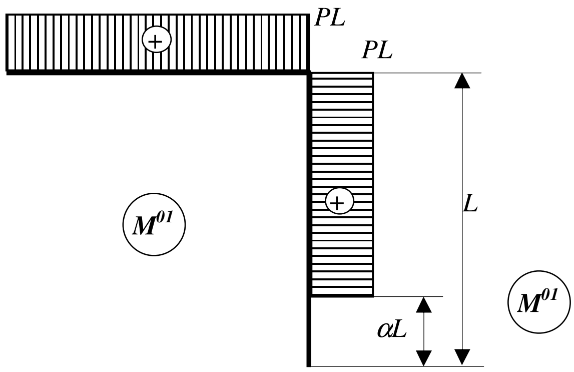

Fig. 6. Moment diagram for N_1.

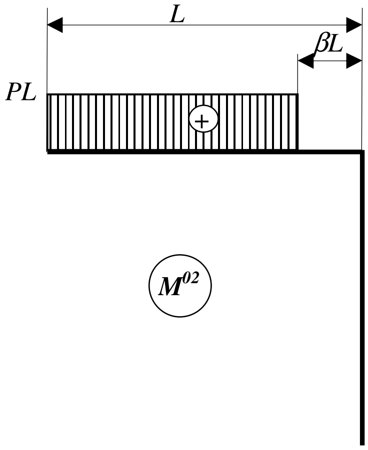

Fig. 7. Moment diagram for N_2.

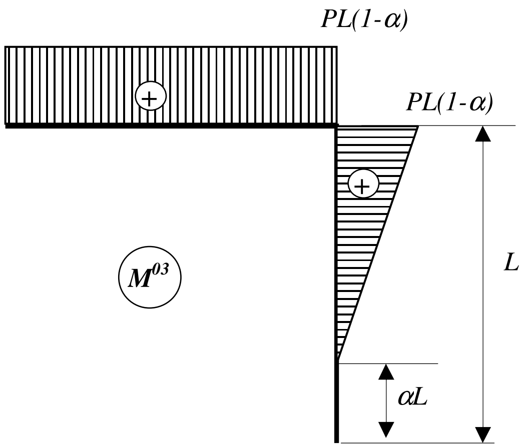

Fig. 8. Moment diagram for P_1.

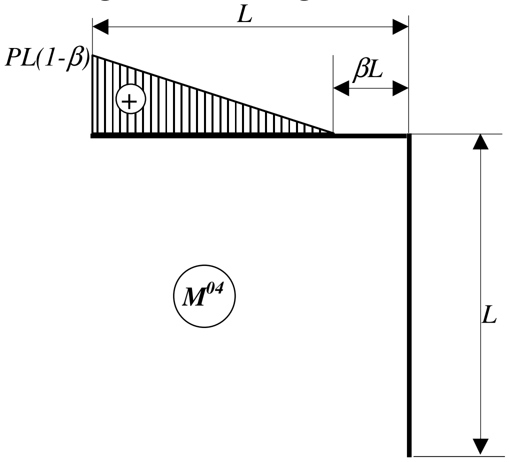

Fig. 9. Moment diagram for P_2.

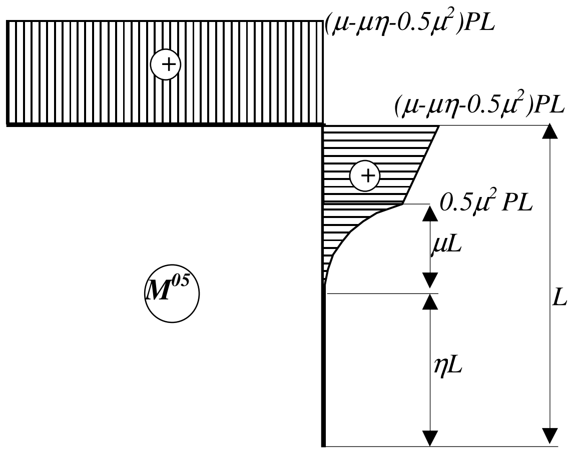

Fig. 10. Moment diagram for q_1.

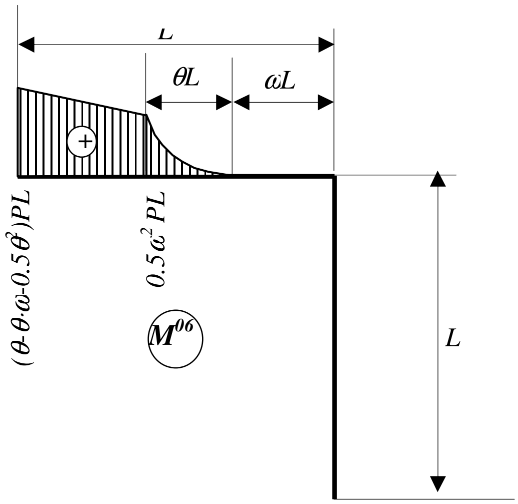

Fig. 11. Moment diagram for q_2

The obtained parametric expressions of the displacements are summarized in Table 3 - Table 5.

Table 3. Parametric expressions of the displacements δ_{10}.

Applied Loads		Parametric expressions for δ_{10}
N_1	α	$\delta_{10}=\frac{3-\alpha^2}{2}\cdot\frac{PL^3}{EI}$
N_2	β	$\delta_{10}=(1-\beta)\cdot\frac{PL^3}{EI}$
P_1	α	$\delta_{10}=\frac{(1-\alpha)(8-\alpha-\alpha^2)}{6}\cdot\frac{PL^3}{EI}$
P_2	β	$\delta_{10}=\frac{(1-\beta)^2}{2}\cdot\frac{PL^3}{EI}$
q_1	η μ	$\delta_{10}=\frac{0.25\mu^4+\eta\cdot\mu^3+\mu^2(1.5\eta^2-4.5)+\mu(\eta^3-9\eta+8)}{6}\cdot\frac{PL^3}{EI}$
q_2	ω θ	$\delta_{10}=\frac{\theta^3-3\theta^2(1-\omega)+3\theta(1-\omega)^2}{6}\cdot\frac{PL^3}{EI}$

Table 4. Parametric expressions of the displacements δ_{20}.

Applied Loads		Parametric expressions for δ_{20}
N_1	α	$\delta_{20}=\frac{1}{2}\cdot\frac{PL^3}{EI}$
N_2	β	$\delta_{20}=\frac{1-\beta^2}{2}\cdot\frac{PL^3}{EI}$
P_1	α	$\delta_{20}=\frac{1-\alpha}{2}\cdot\frac{PL^3}{EI}$
P_2	β	$\delta_{20}=\frac{(1-\beta)^2(2+\beta)}{6}\cdot\frac{PL^3}{EI}$
q_1	η μ	$\delta_{20}=\frac{\mu-\mu\cdot\eta-0.5\mu^2}{2}\cdot\frac{PL^3}{EI}$
q_2	ω θ	$\delta_{20}=\frac{0.25\theta^4+\omega\cdot\theta^3+1.5\cdot\theta^2(\omega^2-1)+\theta(\omega^3-3\omega+2)}{6}\cdot\frac{PL^3}{EI}$

Table 5. Parametric expressions of the displacements δ_{30}.

Applied Loads		Parametric expressions for δ_{30}
N_1	α	$\delta_{30}=(2-\alpha)\frac{PL^2}{EI}$
N_2	β	$\delta_{30}=(1-\beta)\cdot\frac{PL^2}{EI}$
P_1	α	$\delta_{30}=\frac{(1-\alpha)(3-\alpha)}{2}\cdot\frac{PL^2}{EI}$
P_2	β	$\delta_{30}=\frac{(1-\beta)^2}{2}\cdot\frac{PL^2}{EI}$
q_1	η μ	$\delta_{30}=\frac{\mu^3-3\mu^2(2-\eta)+3\mu(1-\mu)(3-\eta)}{6}\cdot\frac{PL^2}{EI}$
q_2	ω θ	$\delta_{30}=\frac{\theta^3-3\theta^2(1-\omega)+3\theta(1-\omega)^2}{6}\cdot\frac{PL^2}{EI}$

In Fig. 12 the hyperstatic frame configuration is represented for the case of the particular values from Table 1.

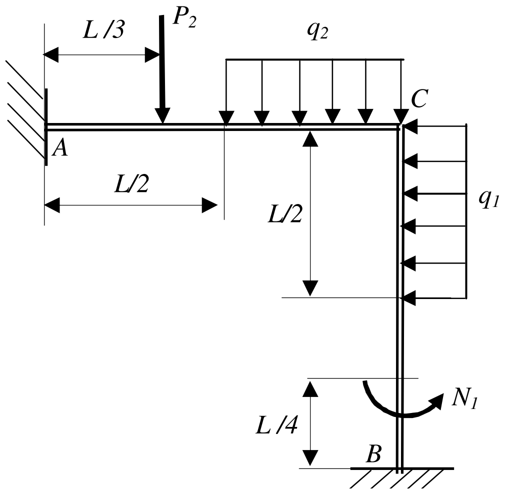

Fig. 12. Particular configuration of the frame.

According to the relations in Table 3 - Table 5, the displacements are:

$$\delta_{10}=\left[\begin{array}{l}\frac{3-\alpha^2}{2}\cdot 1+\frac{(1-\beta)^2}{2}\cdot(-1)+\frac{\theta^3-3\theta^2(1-\omega)+3\theta(1-\omega)^2}{6}\cdot(-2)+\\+\frac{0.25\mu^4+\eta\cdot\mu^3+\mu^2(1.5\eta^2-4.5)+\mu(\eta^3-9\eta+8)}{6}\cdot(-2)\end{array}\right]\frac{PL^3}{EI}$$

$$\delta_{10}=0{,}835069\frac{PL^3}{EI} \quad (3)$$

$$\delta_{20}=\left[\begin{array}{l}\frac{1}{2}\cdot 1+\frac{(1-\beta)^2(2+\beta)}{6}\cdot(-1)+\frac{\mu-\mu\cdot\eta-0.5\mu^2}{2}\cdot(-2)+\\+\frac{0.25\theta^4+\omega\cdot\theta^3+1.5\cdot\theta^2(\omega^2-1)+\theta(\omega^3-3\omega+2)}{6}\cdot(-2)\end{array}\right]\frac{PL^3}{EI}$$

$$\delta_{20}=0{,}112076\frac{PL^3}{EI} \quad (4)$$

$$\delta_{30}=\left[\begin{array}{l}(2-\alpha)\cdot 1+\frac{(1-\beta)^2}{2}\cdot(-1)+\frac{\theta^3-3\theta^2(1-\omega)+3\theta(1-\omega)^2}{6}\cdot(-2)+\\=\frac{\mu^3-3\mu^2(2-\eta)+3\mu(1-\mu)(3-\eta)}{6}\cdot(-2)\end{array}\right]\frac{PL^2}{EI}$$

$$\delta_{30}=1{,}111111\frac{PL^2}{EI} \quad (5)$$

Replacing the obtained values in the equations system (1), the hyperstatic unknowns may be now determined:

$$\begin{cases}X_1=-0{,}758681\cdot P\\X_2=1{,}250579\cdot P\\X_3=-0{,}299190\cdot PL\end{cases} \quad (6)$$

CONCLUSIONS

- Using MATHCAD 14, any hyperstatic systems similar to the presented case can be analyzed, by simply modifying the parameters given in the relations above.
- The parametric relations can also be used for the verification of the results obtained for this type of systems by means of other numerical analysis methods.

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