CONSIDERATIONS REGARDING EVALUATION OF THE ACCURACY ASSESSMENT OF THE ROUNDNESS

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Abstract: *The measurement of parts roundness is a very important for manufacturing industry (for crankshafts, camshafts, bearings and guides measurement). The manner of determining the form deviations influences the accuracy and functionality of the mechanisms.*

This article presents some theoretical considerations on factors influencing the roundness assessment (evaluation method, the number of measurement points on the periphery of the part, distribution of measuring points, eccentricity of the part from rotation axis). The paper presents also a measurement software whose uncertainty will be determined by computerized simulation and experimental methods.

Keywords: *roundness, least squares method, reference circle, minimum zone method,*

1. INTRODUCTION

The manufacturing precision is one of the five factors that influence the quality of the products (manufacturing precision, surfaces quality, manufacturing costs, durability and reliability).

The geometric shape accuracy is one of the four issues of interest regarding the manufacturing accuracy (dimensional precision, accuracy of the geometric shapes, precision of the mutual position of the surfaces, quality of the surfaces).

Dimensional, form and position deviations control is very important because the technological processes gives: assurance of the product quality; information about processes; process control.

The question is how accurate are the results obtained by measuring with a measuring instrument and how the results can be assessde and interpreted.

The round profile is one of the most important fundamental forms of engineering components. Apart from that it is easy to define it (implicitly by the center and radius or explicitly by the circumference), the round form has some advantages such as: symmetry from the axis, can be processed with rotary tools, can be processed with rotary tools, easy assembly becouse of the simetry.

The limit deviations of the geometric shapes should ensure good reliability in operation of the parts in use.

High precision measurement of dimensions, shapes and mutual positions of the surfaces is more and more important requirement in the context of rapidly technology development, to increase products quality, ensure interchangeability in series production, more reliable products, obtaining precision execution machine tools, adjustment of technological processes.

Dimensional measurement technology knows worldwide continuous improvement, especially due to the explosive growth of microelectronics and automation, coordinate measurement techniques and computer-aided geometric shape analysis.

In spite of the obvious benefits of using specialized measurement software, **mathematical models used can be major sources of error in measurement systems**.

Currently there are no accepted standards or methods for assessing the software uncertainty.

2. SOME CONSIDERATIONS ABOUT ROUNDNESS MEASUREMENT

Circularity of a piece depends on its manufacturing or generating methods.
The most common errors associated with round parts are the forms with lobes (odd or even number of lobes). Measuring differences in diameter is not sufficient to measure roundness (figure 1), especially when the number of lobes is not known.

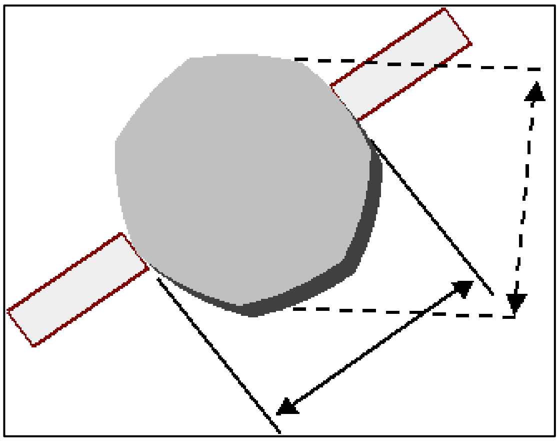
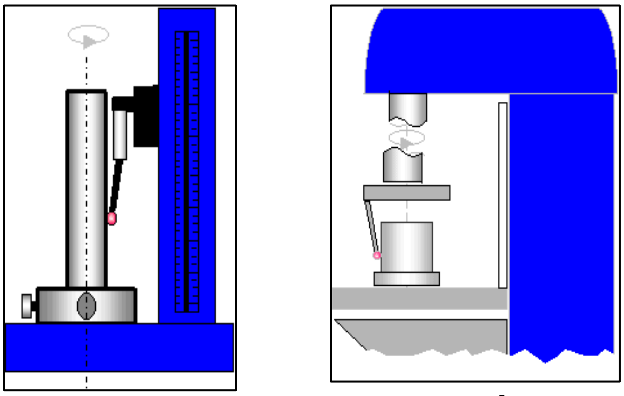


Figure 1. Differential measurement of the diameter

Roundness is usually assessed by rotational techniques by measuring radial deviations from a rotating datum axis. There are two common ways of measuring roundness. One method involves rotation of the part while keeping the measuring transducer fixed (figure 2.a) and the other involves keeping the component fixed while rotating the measuring transducer (figure 2.b).



a. **b.**

Figure 2. Roundness measurement
a. Equipment with rotating table
b. Equipment with rotating stylus

These rotation must be such that its own deviations from a perfect circle with the center on the axis of rotation are negligible compared to the measured profile deviations. The transducer indicates the radial distance "ε" between the contact points on the periphery of the part and a nominal perfect circle.

Parts are not positioned perfectly respect to the center of rotation. For this reason, the value of the transducer includes both, the roundness of the piece and its eccentricity from the center of rotation (figure 3).

For accurate measurement, the measuring system must determine the coordinates of the center of the piece from the center of rotation (in other words, to define the coordinate system of the workpiece relative to the coordinate system of the measuring system).

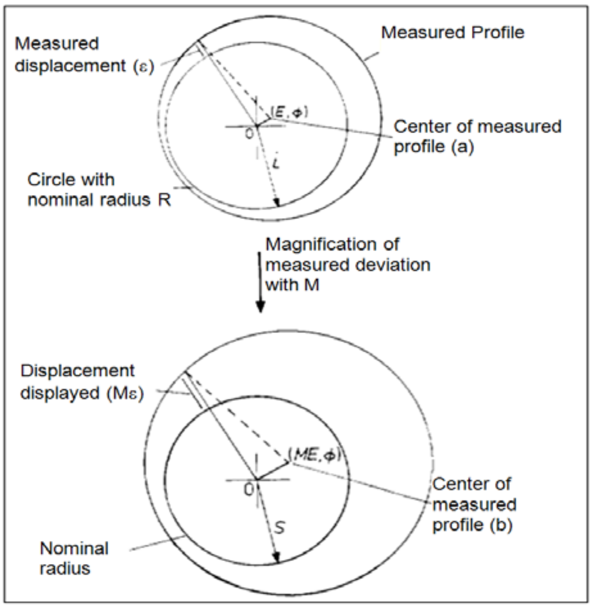


Figura 3. Principles of roundness measurement using radius suppression
(a) instrument coordinates; (b) chart coordinates

The output of the measurement instrument is a polar chart. The computational instruments give also information about out of roundness and eccentricity.

To view the form deviations of the workpiece, the output data of the transducer are amplified and superposed on a convenient nominal circle.

Only the deviation of the displacement transducer is amplified, not radius (this is named radius suppression) [2]. The effects of radius suppression are of fundamental importance to the analysis of errors in the measurement of roundness.

The combination of radius suppression and magnification of the deviations often leads to a visually disconcerting effect on the polar chart. The transformation between instruments coordinates and chart coordinates is such that angles subtended at the origin are preserved. The angular positions of the measured points are not affected by the amplification of the deviations.

The eccentricity, magnification factor and suppression radius are important for the polar chart interpretation.

3. ASSESSMENT OF ROUNDNESS

Assessment of roundness is based on the use of reference futures which are fitted to the data. Form errors of the component are measured relative to these futures.

Four reference figures are internationally accepted for roundness measurement. They are: Least squares circle (LSC); Minimum radial zone circles (MZC); Minimum circumscribed circle (MCC); Maximum inscribed circle (MIC).

The main criteria used for solving the above mentioned assessment methods are:

- Gaussian criterion or criterion of least squares;
- Chebyshev criterion.

Whatever the method of roundness evaluation is used, the following steps are needed:

➤ **First step:** determination of the reference figure center from which to determine the out of roundness;

➤ **The second step:** determination of the reference circle radius;

➤ **The third step:** out of roundness assessment.

The result of out of roundness measurement is influenced by following factors:

- number of measured points;
- distribution of measured points;
- evaluation method.

If measured circular profile is approximated by a minimum number of points (3 points), then criteria for profile fitting are not important. For a more accurate approximation of circular profile are needed a greater number of points**.**

When circular profiles are measured on CMM, the number of measured points can't be very high.

In turning area measurement, the number of 8 and 16 points is inadequate (for all methods - MZCI, LSCI, MCCI and MICI) [3]. Results there are smaller. With higher number of points, roundness varied.

In grinded surfaces, the values measured with 8, 16, 32 and 64 points were smaller than those measured by scanning. Signal filtering in this case does not influence the measurement result (the roundness is similar for all filters).

The results obtained from measurement by scanning are influenced by filters and by fitting methods.

For an accurate profile measurement, the number of points N and the number of estimated lobes Q must have no common factor, and N must be greater than Q [6] .

If N is divisible by Q, then the information acquired by measurement would be limited (see figure 4 and 5).

In conclusion, we can say that for roundness measurement it is appropriate to use scanning and filter data.

Following are the some considerations on fitting methods. For each of the four methods were developed several mathematical models based on which were developed measurement software.

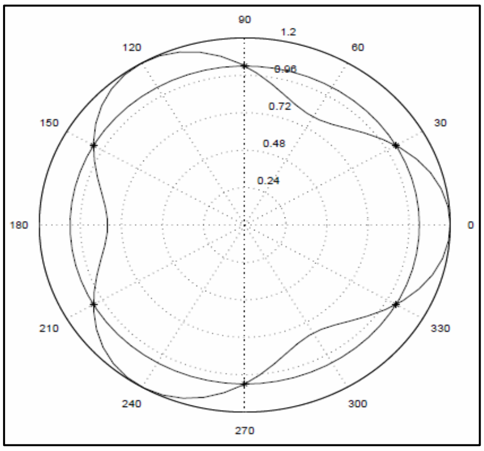


Figure 4. Six uniformly distributed points can not detect three lobes [6]

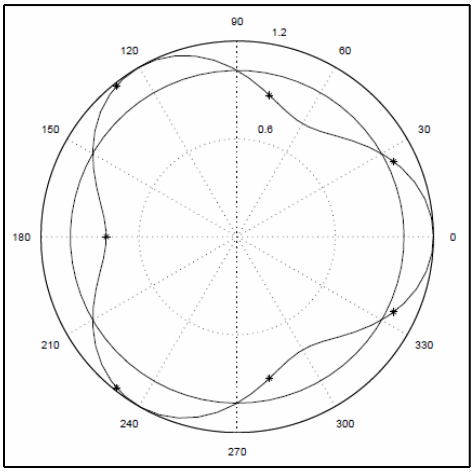


Figure 5. Seven uniformly distributed points allow to detect at least 79% of the amplitude of three lobes [6]

Currently the focus is on evaluating errors introduced by these measurement programs.

In 1988, GIDEP (Government-Industry Data Exchange Program of the U.S.) has published an alert which warned of possible significant measurement uncertainty arising from least-squares fitting software embedded in coordinate measuring machines.

In this report has been analyzed how different measurement techniques, using the same data but different algorithms leading to different results.

GIDEP has shown that for the coordinate measuring machine which use algorithms based on the method of least squares to assess the standard forms (line, plane, circle, sphere, cylinder, cone), the weight of software errors introduced in the total uncertainty of measurement can reach up to 45% (figure 6).

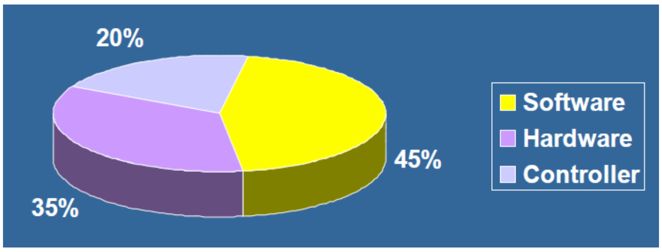


Figure 6. The weight of the factors that influence uncertainty of measurement on CMM [4]

3.1. LEAST SQUARES CIRCLE METHOD (LSC)

The reference element, in relation to which the roundness is determined, is the circle of least squares.

The reference circle center provides information on the profile eccentricity from the center of rotation.

LSC method is based on the average of a number of measured points on the periphery of the profile to determine the parameters of the circle closest to the measured profile.

Mathematically, there are two ways to solve the LSC method:

- **The linear method of least squares**, which solves problems based on systems of linear equations (developed by Gauss and Legendre).
- **The nonlinear least squares method**, which solves the problems based on nonlinear systems of equations. Solving the resulting system of equations is usually done by iterative methods, but at each iteration is used a linearization.

Let a set of N discrete points, describing a circular profile (irregular curve line in figure 7).

The point O_R represents the center of rotation (the origin of the (the origin of the coordinate system of the machine).

The point O_{LSC} represents the center of least squares circle (the origin of the coordinate system of the workpiece).

P is a point on the workpiece profile.

Position of point P in the machine coordinate system is given by angle θ_i and distance R_i. Deviation of profile in point P from the circle of least squares, is Δ.

If we note the distance O_RO_{LSC} (eccentricity of least squares circle from the center of rotation) with "e", can write:

$$e^2 = a^2 + b^2 \quad (1)$$

$$\tan\alpha = b/a \quad (2)$$

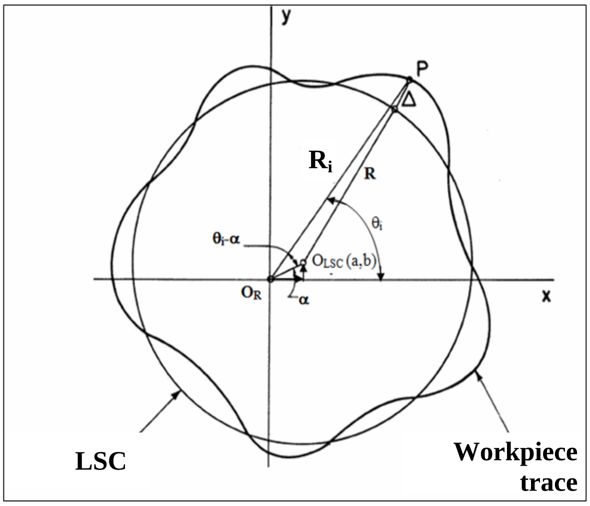


Figure 7. Determination of least squares circle [5]

In triangle O_RPO_{LSC} ew can write:

$$R_i^2 = (R+\Delta)^2 - e^2 + 2R_i \cdot e \cdot \cos(\theta_i - \alpha) = (R+\Delta)^2 - e^2[\sin^2(\theta_i - \alpha) + \cos^2(\theta_i - \alpha)] + 2R_i \cdot e \cdot \cos(\theta_i - \alpha) \quad (3)$$

Result:

$$R_i^2 - 2R_i \cdot e \cdot \cos(\theta_i - \alpha) + e^2 \cos^2(\theta_i - \alpha) = (R+\Delta)^2 - e^2 \sin^2(\theta_i - \alpha) \quad (4)$$

Equation (4) can be written as:

$$R_i = [(R+\Delta)^2 - e^2\sin^2(\theta_i-\alpha)]^{1/2} + e\cos(\theta_i-\alpha) \quad (5)$$

If the piece is well centered to the axis of rotation, "e" is many orders of magnitude smaller than R. In that case, $e^2\sin^2(\theta_i-\alpha)$ can be neglected.

Relation (5) becomes:

$$R_i = R+\Delta + e\cos(\theta_i-\alpha) = R+\Delta + e\cos\theta_i\cos\alpha + e\sin\theta_i\sin\alpha \quad (6)$$

Since: $a = e\cos\alpha$; $b = e\sin\alpha$ the above relation becomes:

$$R_i = R+\Delta + a\cos\theta_i + b\sin\theta_i \quad (7)$$

The relation (7) describe a limaçon [the geometric locus of the points of the plan, which, in polar coordinates (r, θ) satisfy the equation: $r = a\cos\theta \pm b$ sau $r = a\sin\theta \pm b$].

Considering on the circumference of the traced profile a number "n" of points P_i, angular equidistant, defined by the angles θ_i (i=1...n), the sum of squares of the deviations ($E=\sum_{i=1}^{n}\Delta_i^2$) can be minimized by the method of least squares and gives the following estimates parameters:

$$a = \frac{2}{n}\sum_{i=1}^{n} R_i \cos\theta_i$$

$$b = \frac{2}{n}\sum_{i=1}^{n} R_i \sin\theta_i \quad (8)$$

$$R = \frac{1}{n}\sum_{i=1}^{n} R_i$$

Relationship (8) express the parameters "a" and "b" in polar coordinates. They can be expressed in Cartesian coordinates as follows:

$$a = \frac{2}{n}\sum_{i=1}^{n} x_i, \quad i = 1,2,\ldots,n \quad (9)$$

$$b = \frac{2}{n}\sum_{i=1}^{n} y_i, \quad i = 1,2,\ldots,n$$

This approximation is valid under the conditions of a small eccentricities (situation that occurs at the roundness measurement equipments with rotary shaft, when the parts are pretty well centered, but not perfect).

Advantages of the LSC method are:

- Mor stabile. Random noise is filtered out;
- Easy (well researched/ well understood) and fast (has a smaller number of unknown parameters);
- Is less affected by extreme radial coordinates, because the circle of least squares is determined based on all measured points;
- LSC is uniquely determined;
- Allows to change the coordinate system by translation and rotation the measured profile;

Disadvantages of the LSC method are:

- Gives roundness values higher than the MZC (1%...20%);
- Is more difficult to determine by graphical method;

LSC method is recommended when the measurement error is large relative to the form error.

3.2. MINIMUM ZONE CIRCLE METHOD (MZC)

To define the size of the minimum annulus containing all measured points on the periphery of the part, there are two alternative methods: ***minimum area annulus*** or ***minimum width annulus***.

The assessment of the annulus with minimum area that contains a set of points Pi (i = 1, ... n) in plan, measured on the periphery of a circular section is easier to solve, as long as it involves solving a set of linear equations with four unknown (inner radius, outer radius, x and y coordinates of the circle center). These two annulus are not identical.

According the definition, the most accurate value of roundness is given by the minimum width annulus, but it is more difficult to be calculated.

The minimum zone annulus is influenced by the extreme valleys and peaks (figure 8) and by their positions (figure 9) [7] .

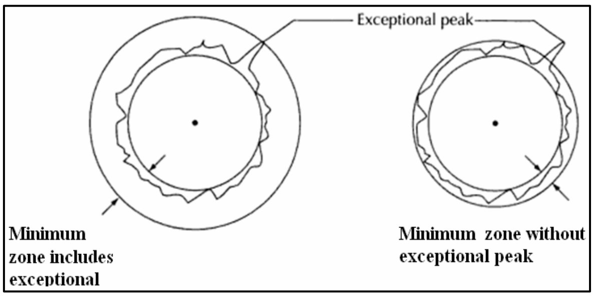


Figure 8. How a single large peak can increase the apparent out of roundness

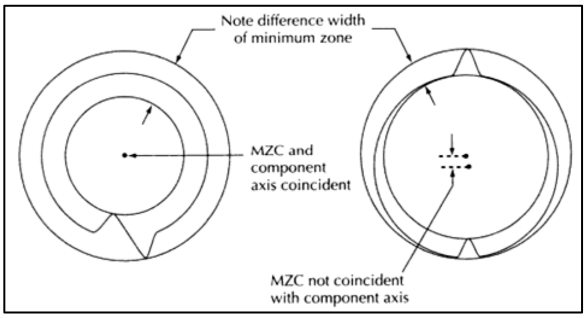
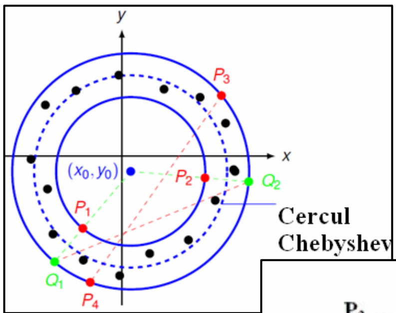


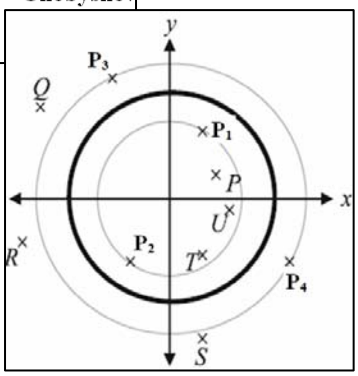
Figure 9. Effect of relative positions of peaks and valley on minimum zone width

The most commonly used methods to solve the minimum Chebyshev problem are: methods based on Simplex algorithm; methods based on “grid" and Euclidean distances; methods based on Fourier analysis.

Iterative method to determine the center coordinates and radii of the circles of the minimal zone is based on the assumption that there are two points on the inner circle and two points on the outer circle, alternating around the center, as in figure 10.



a. circles correctly determined (all points are inside the minimum zone circles)



b. circles incorrectly determined (not all points are inside the minimum zone circles)

Figure 10. The mimimum zone circles

It is considered a finite number of points Pi (xi, yi), i = 1,..., n, describing circular profile in a 2D coordinate system.

R_{max} and R_{min} are the radii of two circles of minimum zone.

$$0 \le R_{min} \le R_{max} \tag{10}$$

The coordinates of the minimum zone circles center are x_0 si y_0 (figure 10.a).

The problem is to determine:

$$\min_{x_0, y_0, R_{min}, R_{max}} (R_{max} - R_{min})$$

for following constraints:

$$\begin{gathered} R_{min} \ge 0 \\ R_{max} - R_{min} \ge 0 \\ (x_i - x_0)^2 + (y_i - y_0)^2 - R_{min}^2 \ge 0, \\ i = 1,\dots,n \\ R_{max}^2 - (x_i - x_0)^2 - (y_i - y_0)^2 \ge 0, \end{gathered} \tag{11}$$

It is considered the circles formed by four points arbitrarily chosen [P1, P2, P3 and P4 (Figure 10.a)], which respect the alternating rule.

If any point P_i satisfies relations (11), then the two circles determine the minimum zone.

If there are points P_i which are outside the ring formed by the two circles [not respected the conditions (11)], then substitute one of the four points with a point outside this area, so that to respect the principles of alternating, and determine the other two circles.

This process is repeated until all points P_i satisfy conditions (11).

Advantages of the MZC method are:

- Give the smallest possible value of the deviation from circularity for a given profile
- Determine the tolerance zone.

Disadvantages of the MZC method are:

- Highly influenced by outliers and random noise.
- Is more difficult to determine by graphical method;
- The MZC solution is not unique; different sets of four points can be determined that have the same MZC width.

MZC method is recommended when the form error is large relative to the measurement error.

MCC and MIC are determined by methods similar to MZC.

3.3. MINIMUM CIRCUMSCRIBED CIRCLE (MCC)

The distance from the point $P_i(x_i,y_i)$ to center ofthe circle is given by:

$$d_i = \sqrt{(x_i - x_o)^2 + (y_i - y_0)^2} > 0 \tag{12}$$

Where: i=1,…,n

x_0, y_0 are the coordinates of the center.

If we consider: $r = \max_{i=1,\dots,n} d_i(x_o, y_0)$ (13)

then: $d_i \le r$ (i=1,...,n)

In this case, all measured data points are within the circle of radius r, centered at (x_0, y_0).

Using the Chebyshev criterion, we require to minimise the radius r by choosing suitable parameters x_0 and y_0 under the constraints:

$$r > 0 \text{ and } r - \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \geq 0,\ i = 1,\ldots,n \qquad (14)$$

There are two possibilities for minimizing the circle in a 2D coordinate system.

➢ **There are 2 contact points (figure 11)**

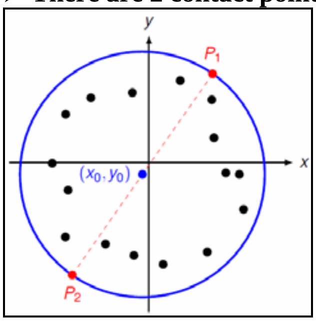


Figure 11. MCC with 2 contact points [8]

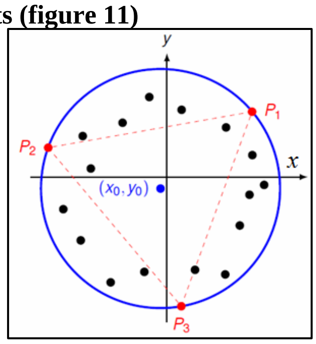


Figure 12. MCC with 2 contact points [8]

MCC is controlled by the points P_1 and P_2.

➢ **There are 3 contact points (figure 12)**

MCC is controlled by the points P_1, P_2 and P_3.

Advantages of the MCC and MCI methods are:

- Are recommended to limit the exact profile of the parts type shafts, respectively holes;
- MCC is uniquely determined;
- Are easily determined by graphical method;

Disadvantages the MCC and MCI methods are:

- Highly influenced by outliers and random noise.
- Can't be used for open circular profie;
- Gives roundness values higher than the LSC;
- MIC is not uniquely determined;

4. MEASUREMENT SOFTWARE

For roundness measurement it is convenient to represent the radial variations as a polar graph. Roundness deviation can be determined by placing a template over the graph. This method is dependent on operator skill and is not very precise.

Therefore, the old template has been replaced with a computer-generated reference circle. Because this circle is derived from the actual measured data, it is possible to mathematically calculate departure of the measured profile from its reference circle.

Based on the above considerations, we design an software for roundness measurement on an equipment with rotary yable

This software is based on least squares measurement method becouse:

- the least square circle and its center are unique;
- ia a robust and fast method;
- is intended for a device that allows a good centering of the workpiece;
- is intended to measure precision parts that are not affected by large form errors;
- the results obtained with this program will be compared with results obtained by measuring on a precision equipment based also on the method of least squares circle.

This software has been designed to satisfy these requirements:

- modular conception to be easily serviced and modified;
- using a mathematical model of trust and robust to ensure a high accuracy of measurement;
- to ensure a high speed of data acquisition and processing;
- to provide the necessary commands for automatic or manual measurement;
  - friendly interface;
  - use filters to isolate frequencies or ranges of UPR to enable detailed examination of individual effects of machining defects and component function.

In the figure 13 is shown the main menu image.

The program allows harmonic analysis of the data, save and print data, and simulating the measurement process.

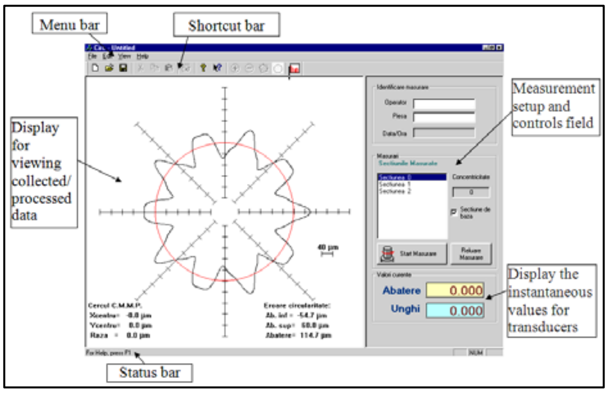


Figure 13. The main menu image

5. FUTURE RESEARCH

Measurement accuracy of the software developed and influence of various factors (number of measured points, piece centering precision, the filter used) will be experimentally determined by comparison with results obtained in similar conditions by measuring with RONCORDER ERG - 11equipment - Japan and by computer simulation method (generating a reference profile for a set of N points with normal distribution and various eccentricities and comparison with results obtained by processing the same set of data by designed software).

Based on these results we will develop a methodology for determining the uncertainty of the roundness measurement equipment (made by INCDMTM) and optimize its performance.

6. CONCLUSIONS:

The most accurate method for determining roundness of a component is to measure the variation of radius from an accurate rotational datum using a scanning probe. The probe remains in contact with the surface and collects a high density of data points. A circle can then be fitted to this data and the roundness calculated.

The final numeric roundness result will be different with the different reference circles applied, so that the users must be sure that the accurately reflects the desired result.

Use of the appropriate reference element is important for measurement roundness and its associated parameters (concentricity, eccentricity, cylindricity, etc.).

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