A NEW APPROACH CONCERNING KINEMATICAL ANALYSIS OF CARDANIC TRANMISSION

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Abstract: *The paper presents a numerical method used for positional kinematical analysis of the cardanic transmission. For this purpose we first determine the differential equations describing the movement of the mechanism in the presence of constraints. These equations are written in the matrix form. Then, the system of differential equations obtained is integrated using numerical integration methods.*

Keywords: *positional kinematical analysis, numerical integration methods, cardanic transmission*

1. INTRODUCTION

The cardanic transmission may be considered as a particular case of spherical quadrilateral mechanism [1]. In the figure below (fig.1) "1" represents the driving element and "3" the driven element. As it can be seen the angle between the driving and the driven element is denoted with "α" and in general is different from zero. The range of this type of transmission is very high. This type of transmission is known under different names such as: universal joint, Hooke coupling, cross cardan [1].

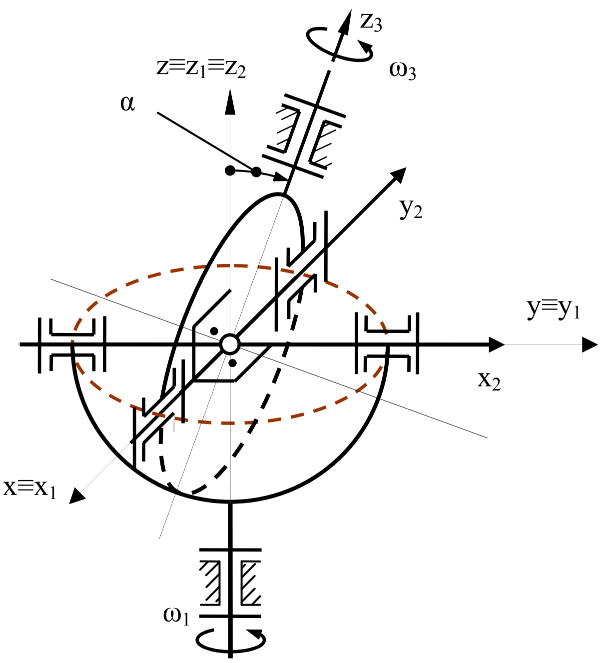

Fig.1 Cardanic Transmission

2. ESTABLISHEMENT THE DIFERENTIAL EQUATIONS OF MOTION IN THE PRESENCE OF CONSTRAINTS

The relationship between kinematical parameters of rigid solid "1" and kinematical parameters of the rigid solid "2" may be written in the matrix form as follows:

$$[R_{10}]\cdot\{\omega_1\} - [R_{20}]\cdot\{\omega_2\} + [R_{20}]\cdot\{\omega_{21}\} = \{0\} \quad (1)$$

In the mathematical relationship (1) the terms involved have the following expressions:

$$[R_{10}] = \begin{bmatrix} \cos(\varphi_1) & -\sin(\varphi_1) & 0 \\ \sin(\varphi_1) & \cos(\varphi_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$[R_{20}] = [R_{10}]\cdot[\Psi_{21}]\cdot[\Theta_{21}] \quad (3)$$

$$[\Psi_{21}] = \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$[\Theta_{21}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{21}) & -\sin(\theta_{21}) \\ 0 & \sin(\theta_{21}) & \cos(\theta_{21}) \end{bmatrix} \quad (5)$$

$$\{\omega_1\} = [\omega_{x_1} \;\vdots\; \omega_{y_1} \;\vdots\; \omega_{z_1}]^T \quad (6)$$

$$\dot{\Phi}_{x_1} = d\Phi_{x_1}/dt = \omega_{x_1} \quad (7)$$

$$\dot{\Phi}_{y_1} = d\Phi_{y_1}/dt = \omega_{y_1} \quad (8)$$

$$\dot{\Phi}_{z_1} = d\Phi_{z_1}/dt = \omega_{z_1} \quad (9)$$

$$\{\omega_{21}\} = [\dot{\theta}_{21} \;\vdots\; 0 \;\vdots\; 0]^T \quad (10)$$

$$\dot{\theta}_{21} = d\theta_{21}/dt \quad (11)$$

The relationship between kinematical parameters of rigid solid "2" and kinematical parameters of the rigid solid "3" may be written in the matrix form as follows:

$$[R_{20}]\cdot\{\omega_2\} - [R_{30}]\cdot\{\omega_3\} + [R_{40}]\cdot\{\omega_{32}\} = \{0\} \quad (12)$$

In the mathematical relationship (12) the terms involved have the following expressions:

$$[R_{30}] = [\Psi_{30}]\cdot[\Theta_{30}]\cdot[\Phi_{30}] \quad (13)$$

$$[\Psi_{30}] = [I_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (14)$$

$$[\Theta_{30}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad (15)$$

$$\alpha = \text{constant} \quad (16)$$

$$[\Phi_{30}] = \begin{bmatrix} \cos(\varphi_3) & -\sin(\varphi_3) & 0 \\ \sin(\varphi_3) & \cos(\varphi_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (17)$$

$$[R_{40}] = [R_{20}] \cdot [\Psi_{32}] \cdot [\Theta_{32}] \quad (18)$$

$$[\Psi_{32}] = \begin{bmatrix} \cos(-\pi/2) & -\sin(-\pi/2) & 0 \\ \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

$$[\Theta_{32}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{32}) & -\sin(\theta_{32}) \\ 0 & \sin(\theta_{32}) & \cos(\theta_{32}) \end{bmatrix} \quad (20)$$

$$\{\omega_2\} = [\omega_{x_2} \; \omega_{y_2} \; \omega_{z_2}]^T \quad (21)$$

$$\dot{\Phi}_{x_2} = d\Phi_{x_2}/dt = \omega_{x_2} \quad (22)$$

$$\dot{\Phi}_{y_2} = d\Phi_{y_2}/dt = \omega_{y_2} \quad (23)$$

$$\dot{\Phi}_{z_2} = d\Phi_{z_2}/dt = \omega_{z_2} \quad (24)$$

$$\{\omega_3\} = [\omega_{x_3} \; \omega_{y_3} \; \omega_{z_3}]^T \quad (25)$$

$$\dot{\Phi}_{x_3} = d\Phi_{x_3}/dt = \omega_{x_3} \quad (26)$$

$$\dot{\Phi}_{y_3} = d\Phi_{y_3}/dt = \omega_{y_3} \quad (27)$$

$$\dot{\Phi}_{z_3} = d\Phi_{z_3}/dt = \omega_{z_3} \quad (28)$$

$$\{\omega_{32}\} = [\dot{\theta}_{32} \; 0 \; 0]^T \quad (29)$$

$$\dot{\theta}_{32} = d\theta_{32}/dt \quad (30)$$

3. INTRODUCING EXTERNAL CONNECTING EQUATIONS

Between rigid solids that make up the system and outside there are certain links that lead to kinematical constraints.

Thus, the links which exist between the rigid solid “1” and outside lead to the following restrictions:

$$\dot{\Phi}_{x_1} = d\Phi_{x_1}/dt = 0 \quad (31)$$

$$\dot{\Phi}_{y_1} = d\Phi_{y_1}/dt = 0 \quad (32)$$

Similarly, the connections between rigid solid “3” and outside determine the following kinematical restrictions:

$$\dot{\Phi}_{x_3} = d\Phi_{x_3}/dt = 0 \quad (33)$$

$$\dot{\Phi}_{y_3} = d\Phi_{y_3}/dt = 0 \quad (34)$$

The relationship between the angle of self-rotation φ_1 and the angle denoted with Φ_{z_1} may be written under differential form as followings:

$$\dot{\varphi}_1 = d\varphi_1/dt = d\Phi_{z_1}/dt = \dot{\Phi}_{z_1} \quad (35)$$

The relationship between the angle of self-rotation φ_3 and the angle denoted with Φ_{z_3} may be written under differential form as followings:

$$\dot{\varphi}_3 = d\varphi_3/dt = d\Phi_{z_3}/dt = \dot{\Phi}_{z_3} \quad (36)$$

First order derivative of the angle denoted with Φ_{z_1} represents the angular speed of the rigid solid “1” which is considered to be constant and known:

$\dot{\Phi}_{z_1} = \omega_{z_1} = \text{constant}$

The unknowns of the problem under consideration are the values of the following quantities:

$\Phi_{x_1}; \Phi_{y_1}; \Phi_{z_1}; \Phi_{x_2}; \Phi_{y_2}; \Phi_{z_2}; \Phi_{x_3}; \Phi_{y_3}; \Phi_{z_3};$
$\theta_{21}; \theta_{32}; \varphi_1; \varphi_3$

4. NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS SYSTEM FOR SOME PARTICULAR CASES

In this chapter we will perform numerical integration of the system of differential equations for one particular case namely that for one specific value of the angle alpha. If the angle is set to $\pi / 3$ [radians] will get the results in the figures below (Fig.1,…., Fig.13)

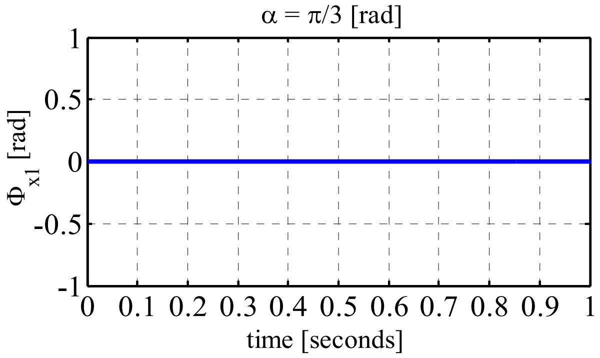

Figure 1. Angle Φ_{x_1} values as function of time

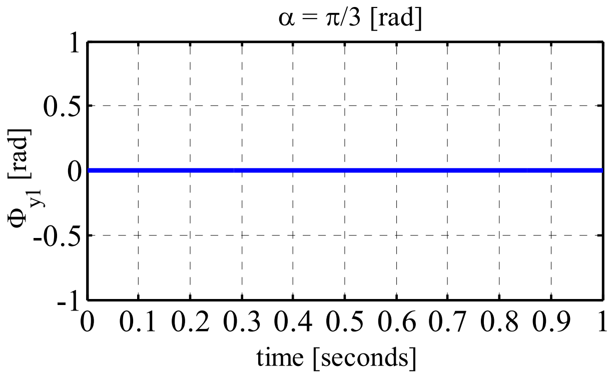

Figure 2. Angle Φ_{y_1} values as function of time

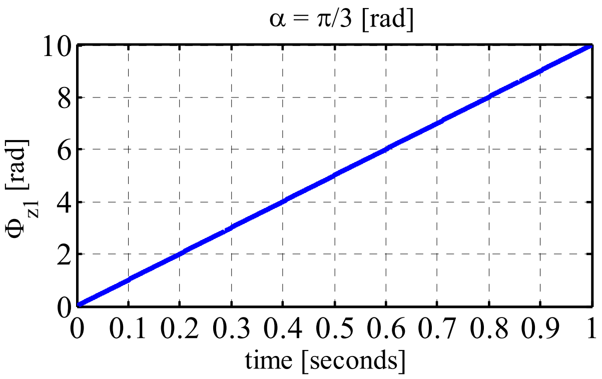

Figure 3. Angle Φ_{z_1} values as function of time

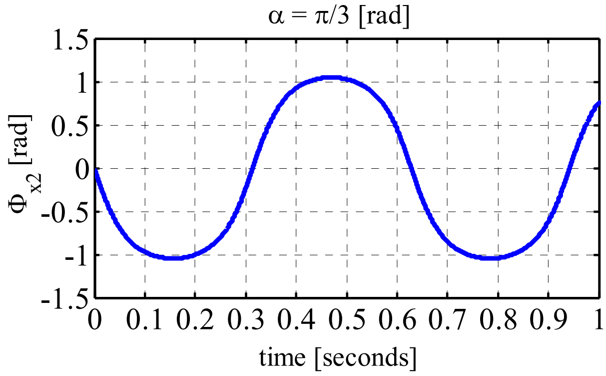

Figure 4. Angle Φ_{x_2} values as function of time

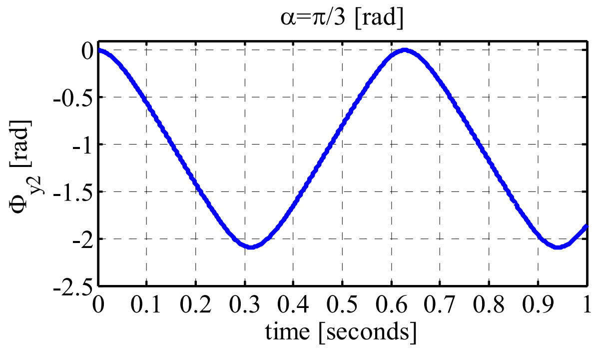

Figure 5. Angle Φ_{y_2} values as function of time

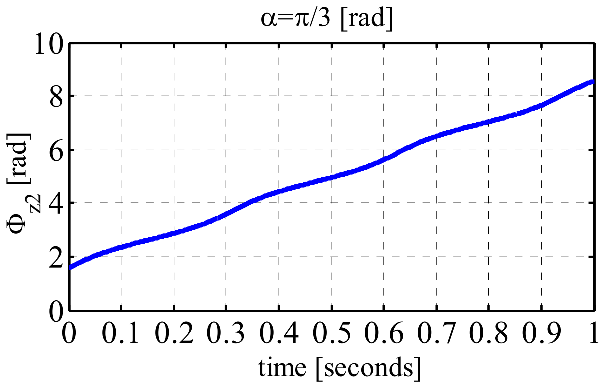

Figure 6. Angle Φ_{z_2} values as function of time

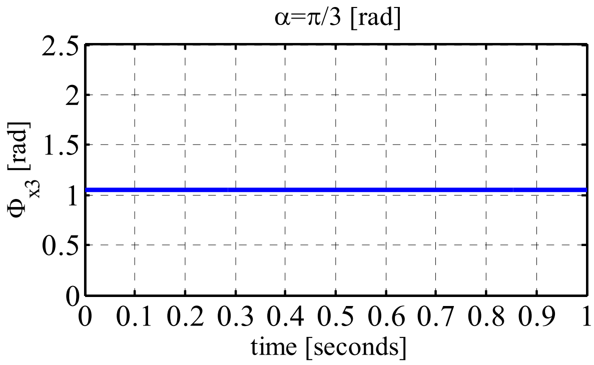

Figure 7. Angle Φ_{x_3} values as function of time

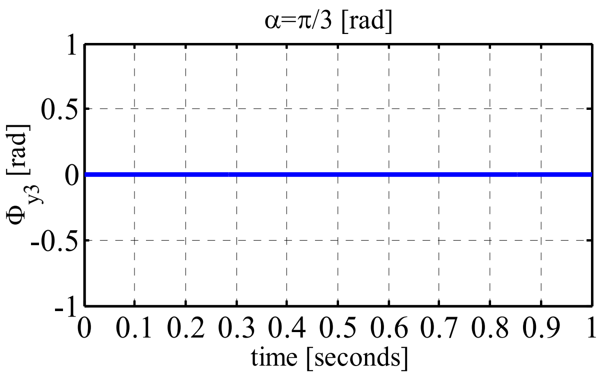

Figure 8. Angle Φ_{y_3} values as function of time

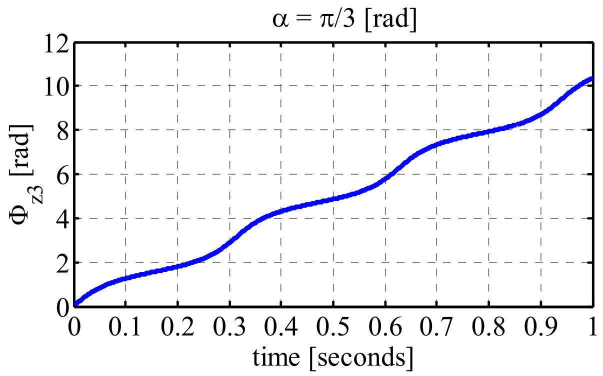

Figure 9. Angle Φ_{z_3} values as function of time

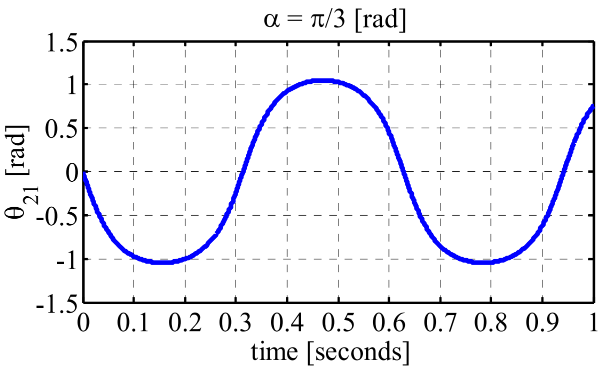

Figure 10. Angle θ_{21} values as function of time

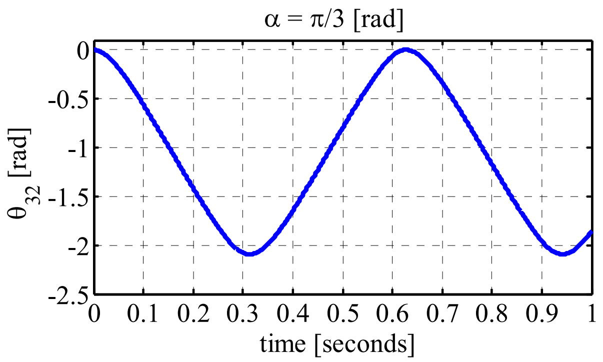

Figure 11. Angle θ_{32} values as function of time

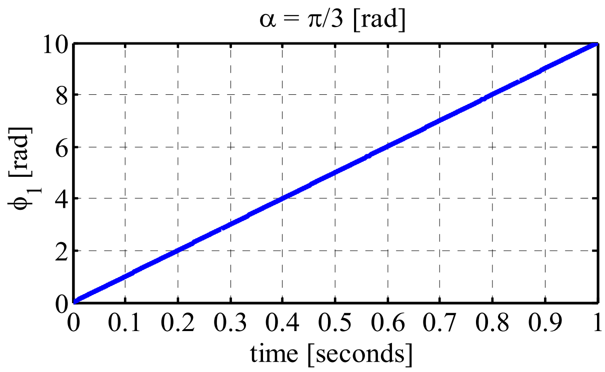

Figure 12. Angle φ_1 values as function of time

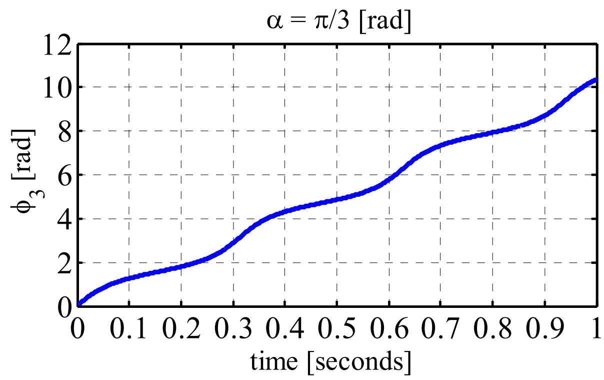

Figure 13. Angle φ_3 values as function of time

5. CONCLUSIONS

The cardanic transmission mechanism the kinematics of which is studied in the present paper is only an example to illustrate the application of the numerical method described in the paper content.

Numerical method presented has a high degree of generality. It can also be applied to any other mechanical system.

When the value of the angle alpha is set to ninety degrees that means $\pi/2$ radians, the mechanism locks.

6. REFERENCES

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