CONSTRUCTION OF COMPLEX 3D SHAPES WITH SUPERELLIPSOIDS AND SUPER CYLINDERS

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Abstract. *This paper presents a CAD study for generating of complex shapes with superellipsoids and super cylinders based on computational geometry. The Madsie Freestyle 1.5.3 application for computation helps in obtaining conclusions referring to shape and profile for different 3D complex objects, but also facilitate the design of new 3D models. Results from this study are applied in geometric constructions and computer aided design used in engineering and sculpture design.*

Keywords: *engineering design, sculpture design, superellipsoid, super cylinder, implicit surface, CAD*

1. INTRODUCTION

Research in engineering and sculpture design involve considerable mathematical modeling to solve a wide variety of problems [1, 2]. A great diversity of geometry representations has been used for reconstruction, modeling, editing, and rendering of 3D objects [3, 4, 5]. The sculptured surfaces, or so-called free-form surfaces, have usually free-formed geometry of complex shapes and are difficult to be machined.

2. SUPERELLIPSOID

In recent years, superellipsoids have received significant attention for object modeling [6].

A superellipsoid, as an ellipsoid's extension, is the result of the spherical product of two 2D models (two superellipses) [6].

A superellipse, analogous to a circle, is expressed as:

$$\left(\frac{x}{a}\right)^{2/\varepsilon} + \left(\frac{y}{b}\right)^{2/\varepsilon} = 1,\ a > 0, b > 0. \qquad (1)$$

and can be written in next form:

$$s(\theta) = \begin{bmatrix} a\cos^{\varepsilon}\theta \\ b\sin^{\varepsilon}\theta \end{bmatrix},\ -\pi \le \theta \le \pi. \qquad (2)$$

where exponentiation with ε is a signed power function such that:

$$\cos^{\varepsilon}\theta = sign(\cos\theta)\left|\cos\theta\right|^{\varepsilon}. \qquad (3)$$

Superellipsoids can be expressed by a spherical product of a pair of such superellipses:

$$r(\eta,\omega) = s_1(\eta)\otimes s_2(\omega) = \begin{bmatrix} \cos^{\varepsilon 1}\eta \\ a_3\sin^{\varepsilon 1}\eta \end{bmatrix} \otimes \begin{bmatrix} a_1\cos^{\varepsilon 2}\omega \\ a_2\sin^{\varepsilon 2}\omega \end{bmatrix} = \\ = \begin{bmatrix} a_1\cos^{\varepsilon 1}\eta\cos^{\varepsilon 2}\omega \\ a_2\cos^{\varepsilon 1}\eta\sin^{\varepsilon 2}\omega \\ a_3\sin^{\varepsilon 1}\eta \end{bmatrix},\ -\frac{\pi}{2} \le \eta \le \frac{\pi}{2};\ -\pi \le \omega \le \pi. \qquad (4)$$

The a_1, a_2, a_3 parameters are scaling factors along the three coordinate axes. ε_1 and ε_2 are derived from the exponents of the two original superellipses.

This flexibility achieved by raising each trigonometric term to an exponent is of particular interest to us. In simple terms, these exponents, control the relative roundness and squareness in both the horizontal and vertical directions.

The shape of the superellipsoid cross section parallel to the [*xoy*] plane is determined by ε_1, while the shape of the superellipsoid cross section in a plane perpendicular to the [*xoy*] plane and containing *z* axis is given by ε_2.

A superellipsoid is defined as the solution of the general form of the implicit equation [7]:

$$\left(\left(\frac{x}{a_1}\right)^{2/\varepsilon_2} + \left(\frac{y}{a_2}\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/\varepsilon_1} + \left(\frac{z}{a_3}\right)^{2/\varepsilon_1} = 1. \qquad (5)$$

All points with coordinates (*x*, *y*, *z*) that correspond to the above equation lie on the surface of the superellipsoid. This is a compact model defined by only five parameters that permits to handle a different shapes. The exponent functions are continuous to ensure that the superellipsoid model deforms continuously and thus has a smooth surface.

This form provides an information on the position of a 3D point related to the superellipsoid surface, that is important for interior/exterior determination [8].

We have an inside-outside function $F(x, y, z)$:

$F(x, y, z) = 1$ when the point lies on the surface;

$F(x, y, z) < 1$ when the point is inside the superellipsoid;

$F(x, y, z) > 1$ when the point is outside.

3. GRAPHICAL REPRESENTATIONS

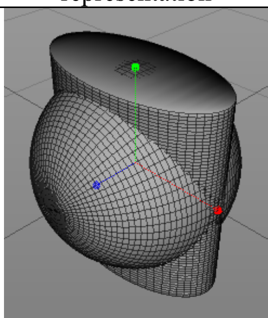
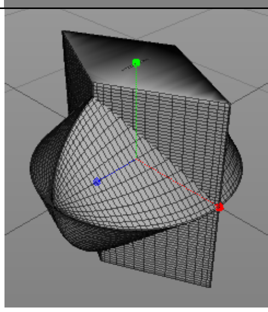
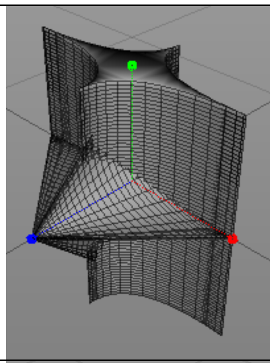
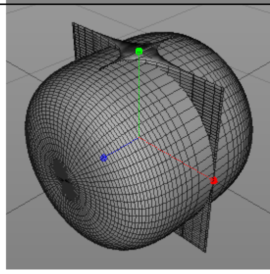
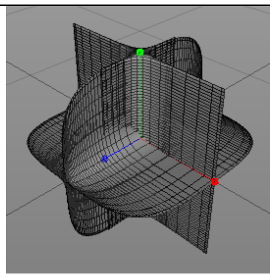
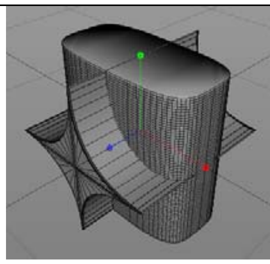
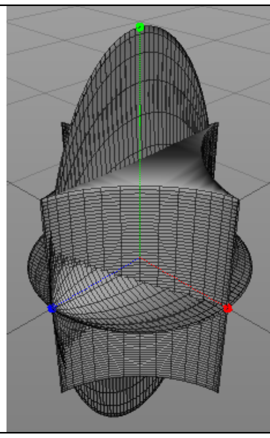
The Madsie Freestyle 1.5.3 application for computation was used to generate the 3D objects with superellipsoids and super cylinders [9] and the graphical representations are given in Table 1.

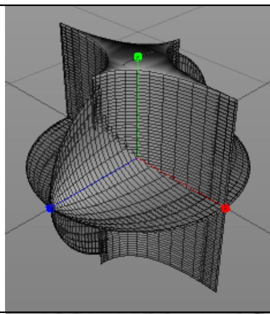
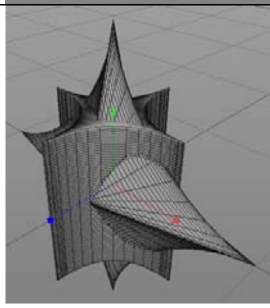
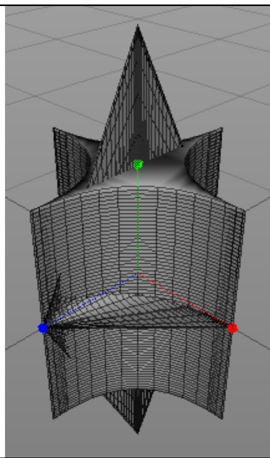
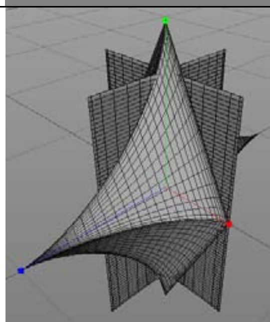
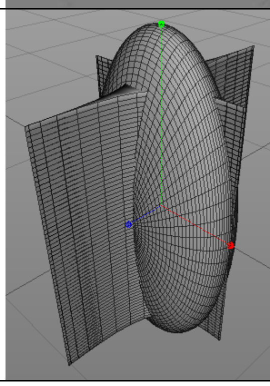
The parameters for superellipsoid are:
Radius X (r_1) - the radius of the ellipsoid along the X-axis;
Radius Y (r_2) - the radius of the ellipsoid along the Y-axis;
Radius Z (r_3) - the radius of the ellipsoid along the Z-axis;
Stacks (n_1) - the number of segments along the Z-axis;
Slices (n_2) - the number of radial segments around the ellipsoid;
Stack Exponent (e_1) - the shape of the ellipsoid;
Slice Exponent (e_2) - the shape of the ellipsoid.

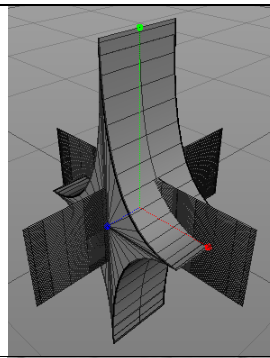
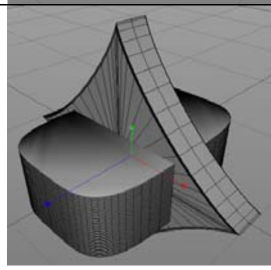
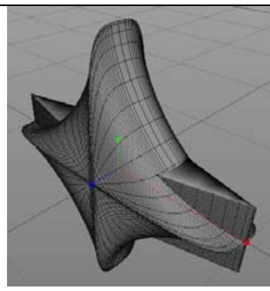
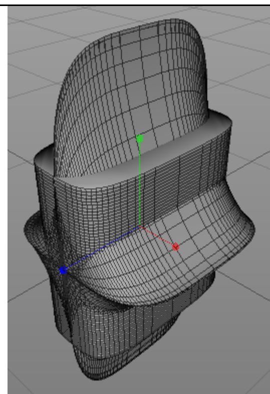
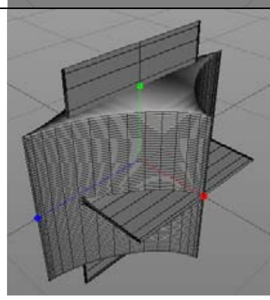
The parameters for super cylinder are:
Radius X (r_4) - the radius of the cylinder along the X-axis;
Radius Z (r_5) - the radius of the cylinder along the Z-axis;
Height (h) - the height of the cylinder;
Stacks (n_3) - the number of segments along the height;
Slices (n_4) - the number of radial segments around the cylinder;
Exponent (e_3) - the shape of the cylinder;
Capped - if enabled will put a polygon at the top and bottom of the cylinder.

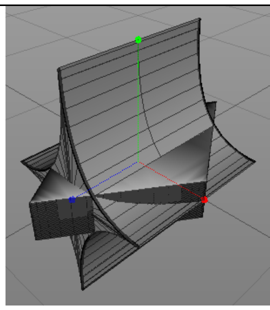
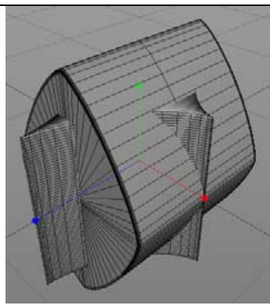
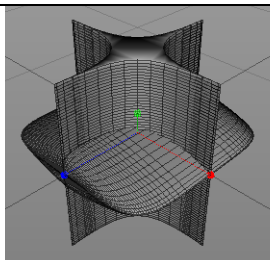
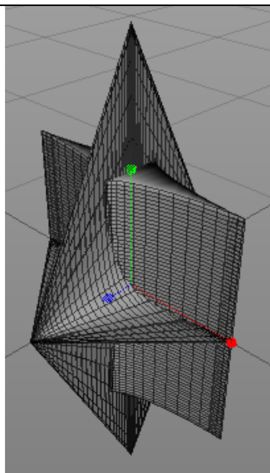
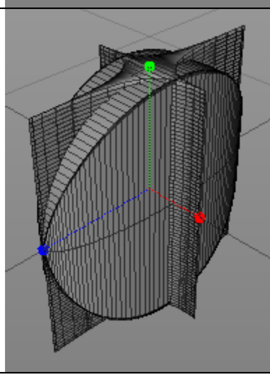
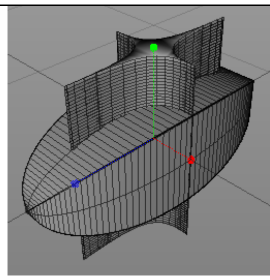
Because there are some numerical issues in computation with both very small or very large values of the exponents, in this study, for safety, they are chosen in the range of 0.01 to about 8.

Table 1. Graphical representations of 3D complex objects

No.	Values of parameters	Axonometric representation
1	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 1; e_2 = 1.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 1;$ Capped - enabled.	
2	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 2; e_2 = 1.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 2;$ Capped - enabled.	
3	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 4; e_2 = 2.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 4;$ Capped - enabled.	
4	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 1; e_2 = 0.5.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 5;$ Capped - enabled.	
5	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 7; e_2 = 0.5.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 7;$ Capped - enabled.	
6	Superellipsoid: $r_1 = 1; r_2 = 1; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 4; e_2 = 0.01.$ Super cylinder: $r_4 = 1; r_5 = 0.5; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 0.5;$ Capped - enabled.	
7	Superellipsoid: $r_1 = 1; r_2 = 2; r_3 = 1;$ $n_1 = 64; n_2 = 64;$ $e_1 = 5; e_2 = 1.$ Super cylinder: $r_4 = 1; r_5 = 1; h = 1;$ $n_3 = 64; n_4 = 64;$ $e_3 = 3;$ Capped - enabled.	

8	Superellipsoid: $r_1 = 1$; $r_2 = 1$; $r_3 = 1$; $n_1 = 64$; $n_2 = 64$; $e_1 = 5$; $e_2 = 1$. Super cylinder: $r_4 = 1$; $r_5 = 0.5$; $h = 1$; $n_3 = 64$; $n_4 = 64$; $e_3 = 4$; Capped - enabled.	
9	Superellipsoid: $r_1 = 2$; $r_2 = 2$; $r_3 = 0.5$; $n_1 = 64$; $n_2 = 64$; $e_1 = 3$; $e_2 = 2$. Super cylinder: $r_4 = 1$; $r_5 = 1$; $h = 1$; $n_3 = 64$; $n_4 = 64$; $e_3 = 4$; Capped - enabled.	
10	Superellipsoid: $r_1 = 1$; $r_2 = 2$; $r_3 = 1$; $n_1 = 64$; $n_2 = 64$; $e_1 = 6$; $e_2 = 2$. Super cylinder: $r_4 = 1$; $r_5 = 1$; $h = 1$; $n_3 = 64$; $n_4 = 64$; $e_3 = 4$; Capped - enabled.	
11	Superellipsoid: $r_1 = 1$; $r_2 = 2$; $r_3 = 2$; $n_1 = 64$; $n_2 = 64$; $e_1 = 2$; $e_2 = 3$. Super cylinder: $r_4 = 1$; $r_5 = 1$; $h = 1.5$; $n_3 = 64$; $n_4 = 64$; $e_3 = 8$; Capped - enabled.	
12	Superellipsoid: $r_1 = 1$; $r_2 = 2$; $r_3 = 0.5$; $n_1 = 64$; $n_2 = 64$; $e_1 = 1$; $e_2 = 1$. Super cylinder: $r_4 = 1$; $r_5 = 1.5$; $h = 1.5$; $n_3 = 64$; $n_4 = 64$; $e_3 = 5$; Capped - enabled.	

13	Superellipsoid: $r_1 = 1$; $r_2 = 2$; $r_3 = 0.5$; $n_1 = 64$; $n_2 = 64$; $e_1 = 4$; $e_2 = 0.01$. Super cylinder: $r_4 = 1.5$; $r_5 = 1.5$; $h = 0.5$; $n_3 = 64$; $n_4 = 64$; $e_3 = 8$; Capped - enabled.	
14	Superellipsoid: $r_1 = 2$; $r_2 = 2$; $r_3 = 0.25$; $n_1 = 64$; $n_2 = 64$; $e_1 = 2.5$; $e_2 = 0.01$. Super cylinder: $r_4 = 1$; $r_5 = 1.5$; $h = 0.5$; $n_3 = 64$; $n_4 = 64$; $e_3 = 0.5$; Capped - enabled.	
15	Superellipsoid: $r_1 = 2$; $r_2 = 2$; $r_3 = 0.5$; $n_1 = 64$; $n_2 = 64$; $e_1 = 3$; $e_2 = 0.5$. Super cylinder: $r_4 = 2$; $r_5 = 0.5$; $h = 0.5$; $n_3 = 64$; $n_4 = 64$; $e_3 = 2$; Capped - enabled.	
16	Superellipsoid: $r_1 = 1$; $r_2 = 2$; $r_3 = 1$; $n_1 = 64$; $n_2 = 64$; $e_1 = 3$; $e_2 = 0.5$. Super cylinder: $r_4 = 0.5$; $r_5 = 1$; $h = 1$; $n_3 = 64$; $n_4 = 64$; $e_3 = 0.5$; Capped - enabled.	
17	Superellipsoid: $r_1 = 1$; $r_2 = 1.5$; $r_3 = 1$; $n_1 = 64$; $n_2 = 64$; $e_1 = 8$; $e_2 = 0.01$. Super cylinder: $r_4 = 1$; $r_5 = 1.5$; $h = 1$; $n_3 = 64$; $n_4 = 64$; $e_3 = 3$; Capped - enabled.	

18	Superellipsoid: r_1 = 1; r_2 = 1,5; r_3 = 1; n_1 = 64; n_2 = 64; e_1 = 4; e_2 = 0.01. Super cylinder: r_4 = 1; r_5 = 1.5; h = 0.25; n_3 = 64; n_4 = 64; e_3 = 2; Capped - enabled.	
19	Superellipsoid: r_1 = 1; r_2 = 1.5; r_3 = 1; n_1 = 64; n_2 = 64; e_1 = 1.5; e_2 = 0.01. Super cylinder: r_4 = 1; r_5 = 1.5; h = 1; n_3 = 64; n_4 = 64; e_3 = 3; Capped - enabled.	
20	Superellipsoid: r_1 = 1; r_2 = 0.25; r_3 = 1; n_1 = 64; n_2 = 64; e_1 = 1.5; e_2 = 0.35. Super cylinder: r_4 = 1; r_5 = 1; h = 1; n_3 = 64; n_4 = 64; e_3 = 5; Capped - enabled.	
21	Superellipsoid: r_1 = 1; r_2 = 2; r_3 = 1; n_1 = 64; n_2 = 64; e_1 = 6; e_2 = 2. Super cylinder: r_4 = 1; r_5 = 0.25; h = 1; n_3 = 64; n_4 = 64; e_3 = 3; Capped - enabled.	
22	Superellipsoid: r_1 = 0.25; r_2 = 1; r_3 = 1; n_1 = 64; n_2 = 64; e_1 = 0.01; e_2 = 1. Super cylinder: r_4 = 0.5; r_5 = 1; h = 1; n_3 = 64; n_4 = 64; e_3 = 5; Capped - enabled.	
23	Superellipsoid: r_1 = 0.5; r_2 = 0.5; r_3 = 1.5; n_1 = 64; n_2 = 64; e_1 = 0.01; e_2 = 1. Super cylinder: r_4 = 0.5; r_5 = 1; h = 1; n_3 = 64; n_4 = 64; e_3 = 5; Capped - enabled.	

4. CONCLUSIONS

This paper presents a CAD method for generation of complex shapes with superellipsoids and super cylinders. The Madsie Freestyle 1.5.3 application for computation helps in obtaining conclusions referring to shape of complex 3D objects.

5. ACKNOWLEDGMENTS

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