

NUMERICAL METHOD USED TO ANALYZE ZERO-ORDER AND THE FIRST ORDER KINEMATICS OF THE CARDANIC TRANSMISSION

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Abstract: The paper presents a numerical method for kinematics analysis of zero order and first order of the cardanic transmission. For this purpose we first determine the differential equations describing the movement of the mechanism. These equations are written in the matrix form. Then, the system of differential equations obtained is solved using numerical integration methods.

Keywords: zero-order kinematics analysis, first order kinematics analysis, numerical integration

1. INTRODUCTION

The cardanic transmission is used to transmit motion and power between two competing motor shafts. The angle between the two motor shafts is different from zero and may vary even during the transmission operation. The range of this type of transmission is very high. This type of transmission is known under different names such as: universal joint, Hooke coupling, cross cardan [1].

It is considered the cardanic transmission in the figure below as presented in [1].

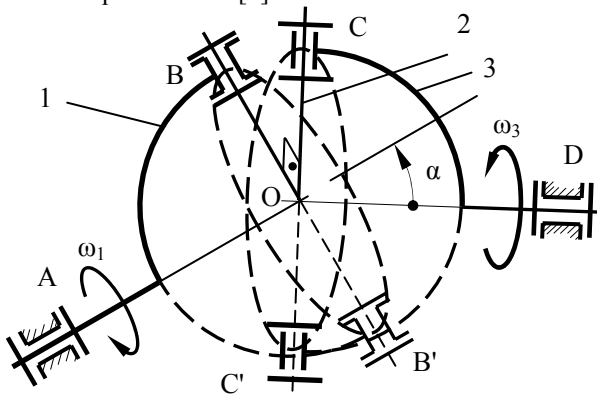


Fig.1 Cardanic Transmission

In the figure above, the relationship between the angular speed of the leading and angular speed of the led element can be written as follows [1]:

$$\omega_3 = \left[\cos \alpha / (1 - (\sin \alpha)^2 \cdot (\cos \varphi_1)^2) \right] \cdot \omega_1 \quad (1)$$

where:

ω_1 - angular velocity of the leading element of the mechanism

ω_3 - angular velocity of the led element of the mechanism

α - angle between elements marked with 1 and 3 (fig.1)

φ_1 - angle of rotation of the element marked with 1 (fig.1)

The analytic relationship between the angular displacement of the leading element and the angular displacement of the led element can be written as follows:

$$\text{tg}(\varphi_3) = \cos(\alpha) \cdot \text{tg}(\varphi_1) \quad (2)$$

In the relationship (1) the following notation is introduced:

$$A = \left[\cos \alpha / (1 - (\sin \alpha)^2 \cdot (\cos \varphi_1)^2) \right] \quad (3)$$

Using the notation expressed by relationship (3), the relationship (1) will be written as follows:

$$\omega_3 = A \cdot \omega_1 \quad (4)$$

Deriving the relationship (4) with respect to time we will obtain:

$$\dot{\omega}_3 = A \cdot \dot{\omega}_1 + \dot{A} \cdot \omega_1 \quad (5)$$

where:

$$\dot{\omega}_1 = d\omega_1/dt \quad (6)$$

$$\dot{A} = dA/dt = B \quad (7)$$

Using the relationship (7) the relation (5) becomes:

$$\dot{\omega}_3 = A \cdot \dot{\omega}_1 + B \cdot \omega_1 \quad (8)$$

In relationship (8), B is given by the following relation:

$$B = - \left[(\sin \alpha)^2 \cos \alpha \cdot \sin(2 \cdot \varphi_1) / (1 - (\sin \alpha)^2 (\cos \varphi_1)^2)^2 \right] \cdot \omega_1 \quad (9)$$

We believe that the angular speed of the leading element is supposed to be a constant so the following relationship may be written:

$$\dot{\omega}_1 = 0 \quad (10)$$

Between the angular displacement of the leading element and its angular velocity we have the following relationship:

$$\dot{\varphi}_1 = \omega_1 \quad (11)$$

Between the angular displacement of the led element and its angular velocity following relationship may be written:

$$\dot{\varphi}_3 = \omega_3 \quad (12)$$

The relationships (8), (10), (11) and (12) form a system of four first order differential equations which may be written in matrix form as follows:

$$[C] \cdot \{\dot{q}\} = [D] \quad (13)$$

where:

$$\{\dot{q}\} = [\dot{\omega}_1 \mid \dot{\omega}_3 \mid \dot{\varphi}_1 \mid \dot{\varphi}_3]^T \quad (14)$$

$$\{q\} = [\omega_1 \mid \omega_3 \mid \varphi_1 \mid \varphi_3]^T \quad (15)$$

In relation (13) [C] represents a square matrix with four lines and four columns which has the following expression:

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \quad (16)$$

$$c_{11} = \cos \alpha / (1 - (\sin \alpha)^2 \cdot (\cos \varphi_1)^2) \quad (17)$$

$$c_{12} = -1 \quad (18)$$

$$c_{13} = c_{14} = 0 \quad (19)$$

$$c_{21} = 1 \quad (20)$$

$$c_{22} = c_{23} = c_{24} = 0 \quad (21)$$

$$c_{31} = c_{32} = c_{34} = 0 \quad (22)$$

$$c_{33} = 1 \quad (23)$$

$$c_{41} = c_{42} = c_{43} = 0 \quad (24)$$

$$c_{44} = 1 \quad (25)$$

In relation (13) [D] represents a column matrix with four lines and one column which has the following expression:

$$[D] = [d_{11} \mid d_{21} \mid d_{31} \mid d_{41}]^T \quad (26)$$

$$d_{11} = \left[\sin \alpha^2 \cos \alpha \sin(2\varphi_1) / (1 - \sin \alpha^2 (\cos \varphi_1)^2)^2 \right] \cdot (\omega_1)^2 \quad (27)$$

$$d_{21} = 0 \quad (28)$$

$$d_{31} = \omega_1 \quad (29)$$

$$d_{41} = \omega_3 \quad (30)$$

The system of differential equations (13) may be written in equivalent for as:

$$\{\dot{q}\} = [C]^{-1} \cdot [D] \quad (31)$$

The system of differential equations (31) can be solved by using numerical integration methods and the values of angular velocities and angular displacements will be obtained.

2. NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS SYSTEM FOR SOME PARTICULAR CASES

In this chapter we will perform numerical integration of the system of differential equations for two particular cases namely that for two specific values of the angle alpha. If the angle is set to $2\pi / 7$ [radians] will get the results in the figures below (Fig.2, Fig.3, Fig.4, Fig.5)

In the figure 2 are represented the values of the angular velocities as function of leading element angular displacements.

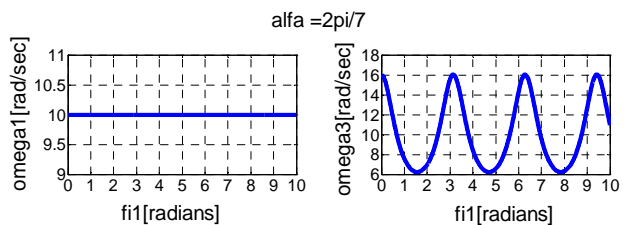


Fig.2 Angular speeds as a function of time

In the figure 3 are represented the values of the angular displacements of the leading and led elements as a function of leading element angular displacements. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

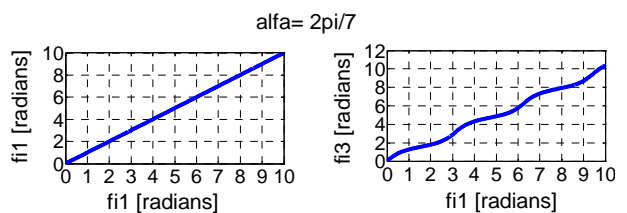


Fig.3 Angular values as a function of leading element angular displacements

In the figure 4 are represented the values of the angular displacements of the leading and led elements as function of time.

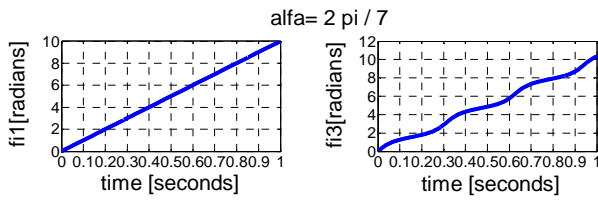


Fig.4 Angular displacements as function of time

In the figure 5 are represented the values of the angular velocities of the leading and led elements as function of time.

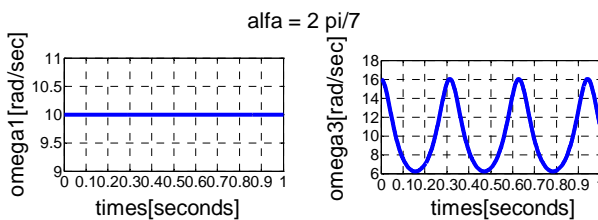


Fig.5 Angular velocities as function of time

If the angle alpha is set to $2\pi/5$ [radians] we will get the results in the figures below (Fig.6, Fig.7, Fig.8, Fig.9)

In the figure 6 are represented the values of the angular displacements of the leading and led elements as a function of leading element angular displacements. The motion is study for a second. The angular velocity of the driving element has the value of ten radians per second.

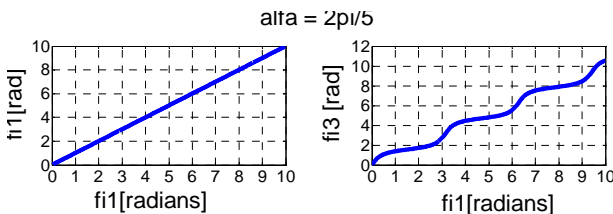


Fig.6 Angular values as a function of leading element angular displacements

In the figure 7 are represented the values of the angular velocities of the leading and led elements as a function of values of the angular displacements of the leading element. In the figure 8 are represented the values of the angular velocities of the leading and led elements as a function of time. The motion is studied for a second. The angular speed of the leading element has the value of ten radians per second. As it can be seen the angular velocity of the driving element (ω_1) is constant.

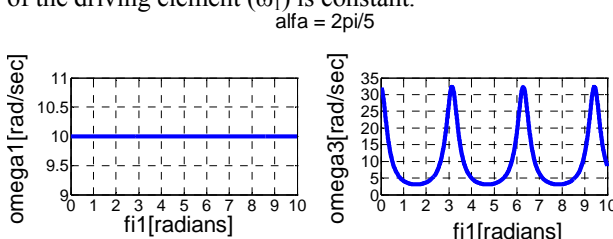


Fig.7 Angular velocities as function of leading elements angular displacements

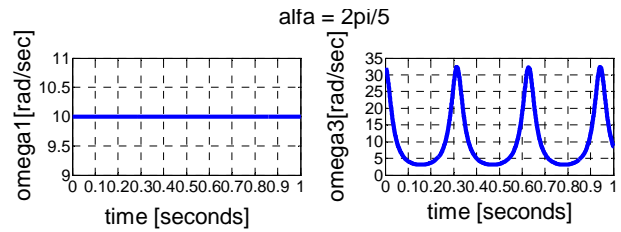


Fig.8 Angular velocities as function of time

In the figure 9 are represented the values of the angular velocities of the leading and led elements as a function of values of time. The motion is study for a second. The angular speed of the leading element has the value of ten radians per second.

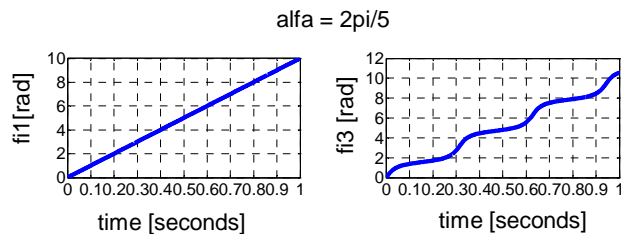


Fig.9 Angular displacements as function of time

If the angle alpha is set to $2\pi/5$ [radians] we will get the results in the figures below (Fig.10, Fig.11, Fig.12, Fig.13)

In the figure 10 are represented the values of the angular displacements of the leading and led elements as a function of time. The motion is study for a second. The angular velocity of the driving element has the value of ten radians per second.

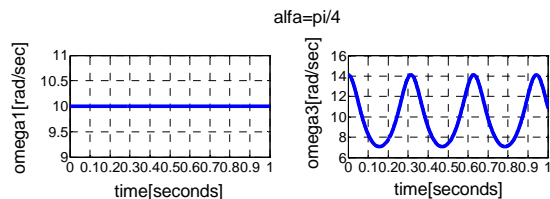


Fig.10 Angular speeds as function of time

In the figure 11 are represented the values of the angular displacements of the leading and led elements of the cardanic transmission as a function of time. The motion of the mechanism is studied for a second. The angular speed of the leading element has the value of ten radians per second. One can easily observe the linear dependence between the values of leading element angular displacements and time.

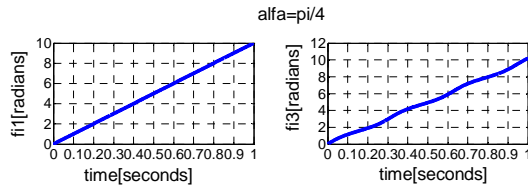


Fig.11 Angular displacements as function of time

In the figure 12 are represented the values of the leading and led elements angular speeds as a function of the values of the leading elements angular displacements.

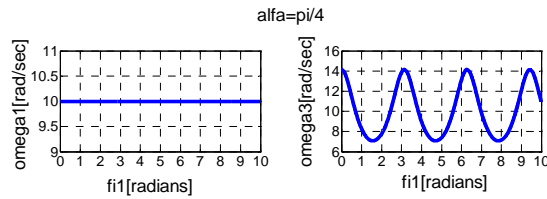


Fig.12 Angular speeds as function of leading element angular displacements

In the figure 13 are represented the values of the leading and led elements angular displacements as a function of the values of the leading elements angular displacements.

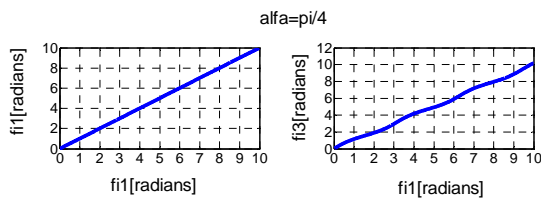


Fig.13 Angular displacements as function of leading elements angular displacements

3. CONCLUSIONS

Analyzing the results it can be seen that the angular speed of the driven element is not constant. It varies between two limits: maximum and minimum.

The values of these two limits are the followings [1]:

$$\omega_{3min.} = \omega_1 \cdot \cos \alpha, \text{ for } \varphi_1 = \pi/2 \quad \varphi_1 = 3\pi/2 \quad (32)$$

$$\omega_{3max.} = \omega_1 / \cos \alpha, \text{ for } \varphi_1 = 0 \quad \varphi_1 = 2 \cdot \pi \quad (33)$$

It can also be noted that the driven element angular speed varies around an average which is given by the following relation:

$$\omega_{3med} = (\omega_{3max} + \omega_{3min}) / 2 \quad (34)$$

or:

$$\omega_{3med} = \omega_1 \cdot \left[1 + (\cos \alpha)^2 \right] / (2 \cos \alpha) \quad (35)$$

The cardanic transmission mechanism the kinematics of which is studied in the present paper is only an example to illustrate the application of the numerical method described in the paper content.

Numerical method presented has a high degree of generality. It can also be applied in other mechanism kinematics study.

Numerical method exposed in the content of this work can be applied to study the kinematics of any other mechanism.

For solving the system of differential equations to be made easier by computer, it is written in matrix form. When the value of the angle alpha is set to ninety degrees that means $\pi/2$ radians, the mechanism locks.

4. REFERENCES

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