NUMERICAL METHOD USED TO ANALYZE ZERO-ORDER AND THE FIRST ORDER KINEMATICS OF THE SWINGING-FORK MECHANISM

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Abstract: *The paper presents a numerical method for kinematics analysis of zero order and first order of the swinging fork mechanism. For this purpose we first determine the differential equations describing the movement of the mechanism. These equations are written in the matrix form. Then, the system of differential equations obtained is solved using numerical integration methods.*

Keywords: zero-order kinematics analysis, first order kinematics analysis, numerical integration

1. INTRODUCTION

The genesis of the swinging fork mechanism can be found in the spherical articulated quadrilateral mechanism [1]. The swinging fork mechanism is used to transmit motion and power between two competing motor shafts. The angle between the driver and the driving element is equal to ninety degrees. One could say that this type of mechanism is a customization of the spherical quadrilateral mechanism. The range of this type of transmission is very high.

2. ESTABLISHMENT THE DIFFERENTIAL EQUATIONS MOTION

Fork oscillating mechanism is considered in the figure below (fig.1) as presented in [1]

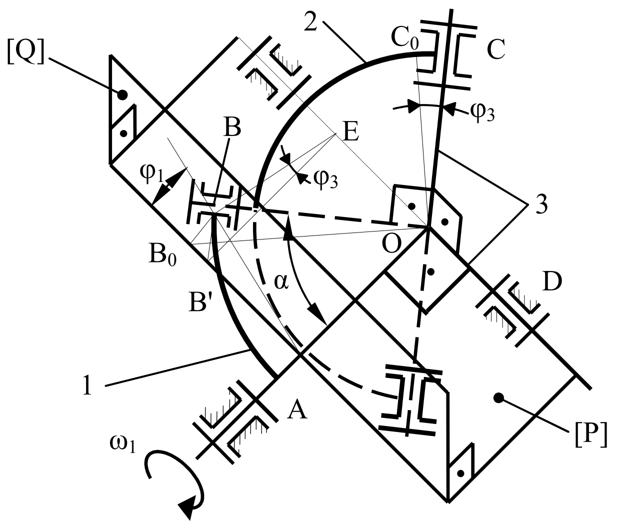

Fig.1 Swinging Fork Mechanism

In the figure above, the relationship between the angular speed of the leading and angular speed of the led element can be written as follows [1]:

$$\omega_3 = \left[\sin(2\alpha)\cdot\cos(\varphi_1)/2\left(1-(\sin\alpha)^2(\cos\varphi_1)^2\right)\right]\cdot\omega_1 \quad (1)$$

where:

ω_1 -angular velocity of the leading element of the mechanism

ω_3 -angular velocity of the led element of the mechanism

α - angle between elements marked with 1 and 3 (Fig.1)

φ_1 - angle of rotation of the element marked with 1(fig.1)

The analytic relationship between the angular displacement of the leading element and the angular displacement of the led element can be written as follows:

$$\mathrm{tg}(\varphi_3) = \mathrm{tg}(\alpha)\cdot\sin(\varphi_1) \quad (2)$$

In the relationship (1) the following notation is introduced:

$$A = \left[\sin(2\alpha)\cdot\cos(\varphi_1)/2\left(1-(\sin\alpha)^2(\cos\varphi_1)^2\right)\right] \quad (3)$$

Using the notation expressed by relationship (2), the relationship (1) will be written as follows:

$$\omega_3 = A\cdot\omega_1 \quad (4)$$

Deriving the relationship (4) with respect to time we will obtain:

$$\dot{\omega}_3 = A\cdot\dot{\omega}_1 + \dot{A}\cdot\omega_1 \quad (5)$$

where:

$$\dot{\omega}_1 = \mathrm{d}\omega_1/\mathrm{d}t \quad (6)$$

$$\dot{A} = \mathrm{d}A/\mathrm{d}t = B \quad (7)$$

Using the relationship (7) the relation (5) becomes:

$$\dot{\omega}_3 = A\cdot\dot{\omega}_1 + B\cdot\omega_1 \quad (8)$$

In relationship (8), B is given by the following relation:

$$B = -(B_1/B_2)\cdot\omega_1 \tag{9}$$

where:

$$B_1 = \sin(2\alpha)\cdot\sin(\varphi_1)\cdot\left[1+\sin(\alpha)^2\cdot\cos(\varphi_1)^2\right] \tag{10}$$

$$B_2 = 2\cdot\left[1-\sin(\alpha)^2\cdot\cos(\varphi_1)^2\right]^2 \tag{11}$$

We believe that the angular speed of the leading element is supposed to be a constant so the following relationship may be written:

$$\dot{\omega}_1 = 0 \tag{12}$$

Between the angular displacement of the leading element and its angular velocity we have the following relationship:

$$\dot{\varphi}_1 = \omega_1 \tag{13}$$

Between the angular displacement of the led element and its angular velocity following relationship may be written:

$$\dot{\varphi}_3 = \omega_3 \tag{14}$$

The relationships (8), (12), (13) and (14) form a system of four first order differential equations which may be written in matrix form as follows:

$$[\mathrm{C}]\cdot\{\dot{q}\} = [\mathrm{D}] \tag{15}$$

where:

$$\{\dot{q}\} = [\dot{\omega}_1 \mid \dot{\omega}_3 \mid \dot{\varphi}_1 \mid \dot{\varphi}_3]^{\mathrm{T}} \tag{16}$$

$$\{q\} = [\omega_1 \mid \omega_3 \mid \varphi_1 \mid \varphi_3]^{\mathrm{T}} \tag{17}$$

In relation (15) [C] represents a square matrix with four lines and four columns which has the following expression:

$$[\mathrm{C}] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \tag{18}$$

$$c_{11} = \left[\sin(2\alpha)\cdot\cos(\varphi_1)/2\left(1-(\sin\alpha)^2(\cos\varphi_1)^2\right)\right] \tag{19}$$

$$c_{12} = -1 \tag{20}$$

$$c_{13} = c_{14} = 0 \tag{21}$$

$$c_{21} = 1 \tag{22}$$

$$c_{22} = c_{23} = c_{24} = 0 \tag{23}$$

$$c_{31} = c_{32} = c_{34} = 0 \tag{24}$$

$$c_{33} = 1 \tag{25}$$

$$c_{41} = c_{42} = c_{43} = 0 \tag{26}$$

$$c_{44} = 1 \tag{27}$$

In relation (15) [D] represents a column matrix with four lines and one column which has the following expression:

$$[\mathrm{D}] = [d_{11} \mid d_{21} \mid d_{31} \mid d_{41}]^{\mathrm{T}} \tag{28}$$

$$d_{11} = (B_1/B_2)\cdot(\omega_1)^2 \tag{29}$$

$$b_{21} = 0 \tag{30}$$

$$b_{31} = \omega_1 \tag{31}$$

$$b_{41} = \omega_3 \tag{32}$$

The system of differential equations (15) may be written in equivalent for as:

$$\{\dot{q}\} = [\mathrm{C}]^{-1}\cdot[\mathrm{D}] \tag{33}$$

The system of differential equations (33) can be solved by using numerical integration methods and the values of angular velocities and angular displacements will be obtained.

3. NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS SYSTEM FOR SOME PARTICULAR CASES

In this chapter we will perform numerical integration of the system of differential equations for two particular cases namely that for two specific values of the angle alpha.
If the angle is set to forty-five degrees will get the results in the figure below (Fig.2, Fig.3, Fig.4, Fig.5)
In the figure 2 are represented the values of the angular velocities of the leading and the led element as function of time. The movement is studied for a second. The angular velocity of the leading element has the value of ten radians per second. We can very easily observe that the angular velocity of the driven element is not constant. It is oscillating between two extreme values: minimum and maximum.

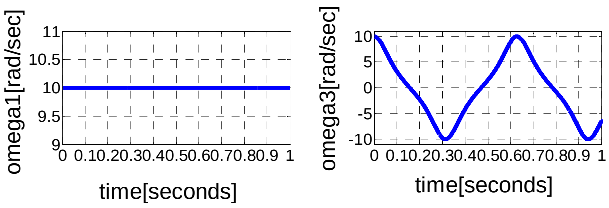

Fig.2 Angular speeds as a function of time

In the figure 3 are represented the values of the angular displacements of the leading and led elements as a function of time. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

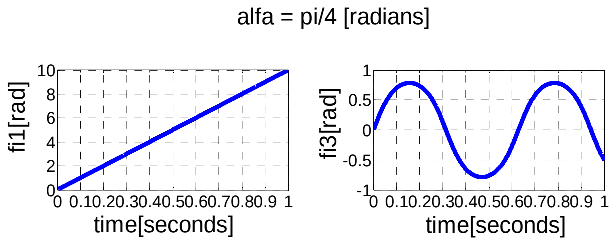

Fig.3 Angular values as a function of time

In the figure 4 are represented the values of the angular velocities as function of leading element angular displacements.

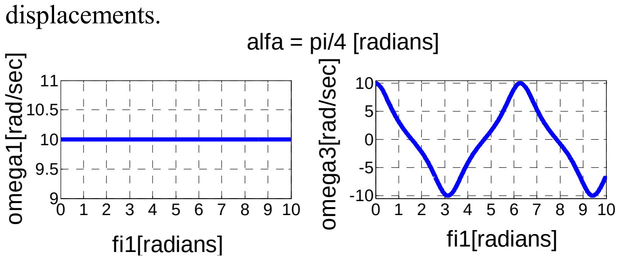

Fig.4 Angular speeds as function of leading element angle

In the figure5 are represented the values of the angular displacements of the leading and led elements as a function of leading element angular displacements.

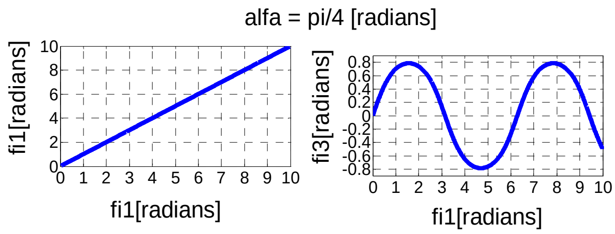

Fig.5 Angular displacements as a function of leading element angle

If the angle alpha is set to $2\pi/7$ radians, we will get the results in the figures below (Fig.6, Fig.7, Fig.8, Fig. 9).
In the figure 6 are represented the values of the angular velocities of the leading and led elements as a function of values of the angular displacements of the leading element As it can be seen the angular velocity of the driving element (ω_1) is constant. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

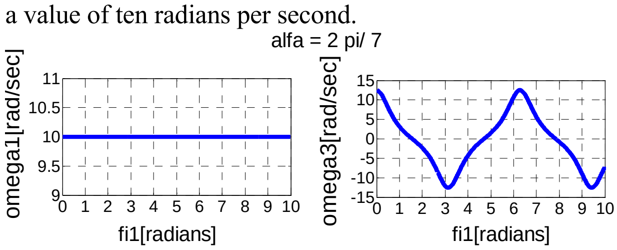

Fig.6 Angular speeds as function of leading element angle

In the figure 7 are represented the values of the angular displacements of the leading and led elements as a function of values of the angular displacements of the leading element.

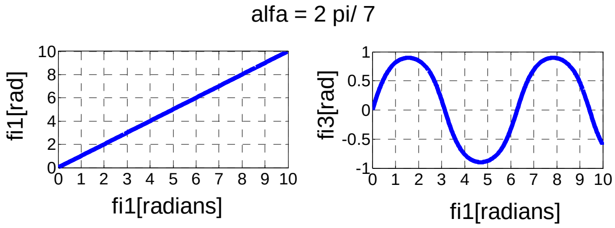

Fig.7 Angular displacements as a function of leading element angle

In the figure 8 are represented the values of the angular velocities of the leading and led elements as a function of time. As it can be seen the angular velocity of the driving element (ω_1) is constant. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

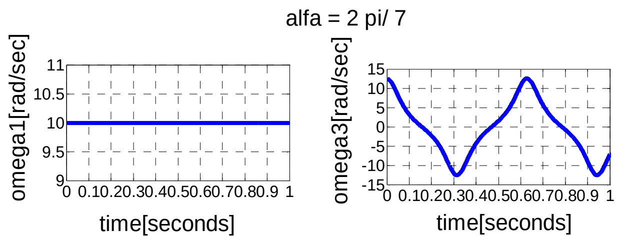

Fig.8 Angular speeds as a function of time

In the figure 9 are represented the values of the angular displacements of the leading and led elements as a function of time. The motion is study for a second. The angular velocity of the driving element has the value of ten radians per second.

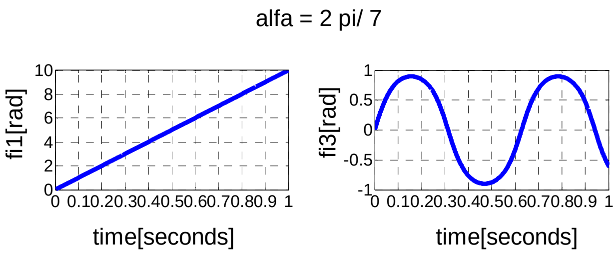

Fig.9 Angular values as a function of time

If the angle alpha is set to $\pi/6$ radians, will get the results in the figures below (Fig.10, Fig.11, Fig.12, Fig.13).
In the figure 10 are represented the values of the angular velocities of the leading and led elements as a function of time. As it can be seen the angular velocity of the driving element (ω_1) is constant. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

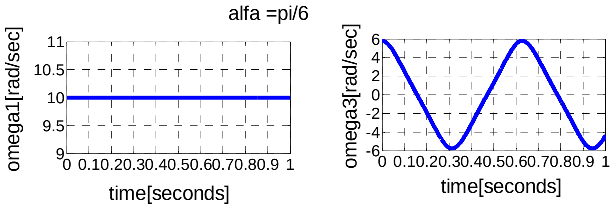

Fig.10 Angular velocities as a function of time

In the figure 11 are represented the values of the angular displacements of the leading and led elements as a function of time. As it can be seen the angular velocity of the driving element (ω_1) is constant. The motion is study for a second. The angular velocity of the driving element has a value of ten radians per second.

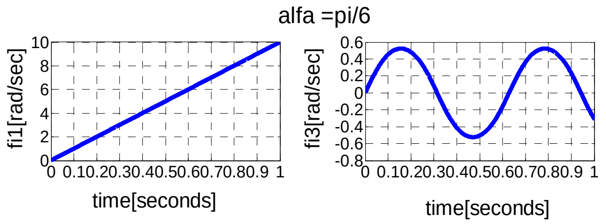

Fig.11 Angular displacements as function of time

In the figure 12 are represented the values of the angular velocities as function of leading element angular displacements.

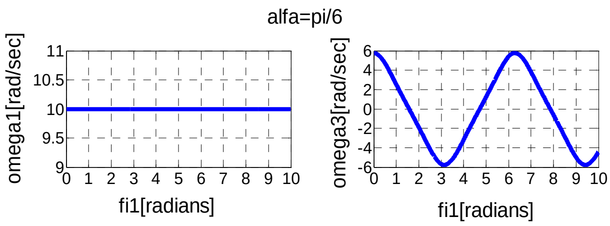

Fig.12 Angular velocities as function of angular displacements of the leading element

In the figure 13 are represented the values of the angular displacements of the leading and led elements as a function of values of the angular displacements of the leading element.

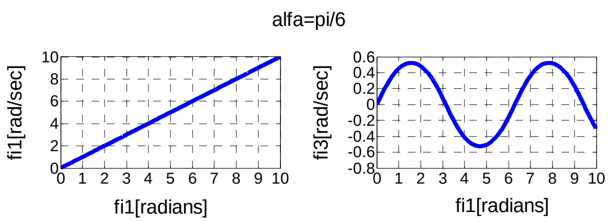

Fig.13 Angular displacements as function of angular displacements of the leading element

4. CONCLUSIONS

Analyzing the results it can be seen that the angular speed of the driven element is not constant. It varies between two limits: maximum and minimum.

The values of these two limits are the followings [1]:

$$\omega_{3\max.} = \omega_1 \cdot tg\alpha \text{, for } \varphi_1 = 0 \tag{34}$$

$$\omega_{3\min.} = -\omega_1 \cdot tg\alpha \text{, for } \varphi_1 = \pi \tag{35}$$

It can also be noted that the driven element has an oscillating motion. For this reason the mechanism studied in this work is called oscillating fork mechanism.

The fork oscillating mechanism the kinematics of which is studied in the present paper is only an example to illustrate the application of the numerical method described in the paper content.

Numerical method presented has a high degree of generality. It can also be applied in other mechanism kinematics study.

Numerical method exposed in the content of this work can be applied to study the kinematics of any other mechanism.

For solving the system of differential equations to be made easier by computer, it is written in matrix form.

When the angle alpha is set ninety degrees the mechanism locks.

5. REFERENCES

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